Maths – GCSE to A Level

Summer Work

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Linear Equations

An equation that only includes variables with a power of 1 is a *linear equation*. All linear equations will have graphs that form a straight line.

Solving Linear Equations:

Always ensure that you show your steps when solving any equations.

- Example 1:

Solve $3x + 7 = 43$

-2 -2

$3x = 36$

÷3 ÷3

$x = 12$

Minimum methods acceptable: As e.g. with no arrows

- Example 2:

Solve $\frac{5x-2}{7} = 9$

To help decide the order: imagine secret brackets around the numerator of the fraction. $\frac{(5x-2)}{7}$

×7 ×7

$5x - 2 = 63$

+2 +2

$5x = 65$

+2 +2

$x = 13$

Minimum methods acceptable: As e.g. with no arrows

- Example 3:

Solve $2x + 4 = 7x - 11$

Collect x terms first. Move the SMALLEST amount of x's to the biggest

-2x -2x

$4 = 5x - 11$

+11 +11

$15 = 5x$

÷3 ÷3

$3 = x$

Minimum methods acceptable: As e.g. with no arrows

Exercise 1:

1) Solve the following equations. Write your solution as a fraction when required.

a) $11q - 17 = 60$
b) $6p + 10 = 52$
c) $8y - 1 = 0$
d) $4 + 5n = 64$
e) $13 - b = 7$
f) $20 - 3c = 8$

2) Solve these equations involving divisions. Write your solution as a fraction when required.

a) $\frac{x}{2} - 5 = 4$
b) $3 + \frac{x}{4} = 10$
c) $\frac{x+3}{4} = 12$
d) $\frac{5x}{6} = \frac{1}{4}$

3) Solve these equations involving unknowns on both sides. Write your solution as a fraction when required.

a) $5m + 6 = 3m + 12$
b) $2p + 4 = p - 3$
c) $5q - 4 = 3 - q$
d) $7 - 3x = 5 - 2x$

4) Solve these equations involving brackets. Write your solution as a fraction when required.

a) $e + 3(e + 1) = 2e$
b) $5(f + 6) = 35f$
c) $3(2g + 1) + 2(g - 1) = 23$
d) $5h - 3(h - 1) = 39$

5) Solve these more complex equations. Write your solution as a fraction when required.

a) $\frac{21}{x} = 7$
b) $30 = \frac{6}{y}$
c) $\frac{2x-1}{3} = \frac{x}{2}$
d) $\frac{12}{2x-3} = 4$
e) $\frac{6}{x} - 3 = 7$
f) $\frac{5}{x+5} = \frac{15}{x+7}$

Now check your work, answers on page 18.

Quadratic Equations

An equation that only includes variables with a power of 2 is a *quadratic equation*. All quadratic equations will have graphs that form ∪ or ∩ shaped parabola.

Factorising Quadratic Expressions:

Example 1:

a) Factorise $x^2 + 5x - 24$

$x^2 + 5x - 24$

Product = -24 Sum = +5

-4 × 6 = 2 ×

-6 × 4 = -2 ×

-3 × 8 = +5 ✓

Solution: $(x - 3)(x + 8)$

b) Factorise $x^2 - 49$

$x^2 + 0x - 49$

Product = -49 Sum = +0

-7 × 7 = 0 ✓

Solution: $(x - 7)(x + 7)$

c) Factorise $6x^2 - x - 2$

$6x^2 - 1x - 2$

6 × -2

Product = -12 Sum = -1

-4 × 3 = -1 ✓

Separate x: $6x^2 - 4x + 3x - 2$

Factorise pairs: $2x(3x - 2) + 1(3x - 2)$

Solution: $(2x + 1)(3x - 2)$

Example 2:

Hence solve a) $x^2 + 5x - 24 = 0$

$\therefore (x - 3)(x + 8) = 0$

So $x - 3 = 0$ or $x + 8 = 0$

$x = 3$ or $x = -8$

b) $x^2 - 49 = 0$

$\therefore (x - 7)(x + 7) = 0$

So $x - 7 = 0$ or $x + 7 = 0$

$x = 7$ or $x = -7$

c) $6x^2 - x - 2 = 0$

$\therefore (2x + 1)(3x - 2) = 0$

So $2x + 1 = 0$ or $3x - 2 = 0$

$x = -\frac{1}{2}$ or $x = \frac{2}{3}$

Completing the Square:

Example 3: Write in the form $(x + a)^2 + b = 0$ and hence solve $x^2 + 6x - 16 = 0$

Compare $(x + a)^2 = x^2 + 2ax + a^2$ with the 1st two terms of the equation $x^2 + 6x$

$\therefore 2a = 6$

$a = 3$

Remove extra a^2 term

Include 3rd term of original equation

Substitute into required form: $(x + 3)^2 - 3^2 - 16 = 0$

Simplify: $(x + 3)^2 - 25 = 0$ (Completed Square Form)

Rearrange to solve: $(x + 3)^2 = 25$

$(x + 3) = \pm 5$

$x = -3 \pm 5$ $\therefore x = 2$ or $x = -8$

Using the Quadratic Formula:

REMEMBER
A quadratic equation MUST equal ZERO before you solve it.

Example 4:

Use the quadratic formula to solve $3x^2 - 7x + 2 = 0$

Substitute values from $ax^2 + bx + c = 0$ into $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{7 \pm \sqrt{49 - 24}}{6} = \frac{7 \pm \sqrt{25}}{6} = \frac{7 \pm 5}{6} = \frac{1}{3} \text{ or } 2$$

Exercise 2:

1) Factorise the following quadratic expressions:

a) $x^2 - 5x + 4$
b) $x^2 + 7x + 10$
c) $x^2 - 2x - 15$
d) $5y + 6 + y^2$
e) $10 - 11x + x^2$
f) $y^2 - y - 12$
g) $x^2 - 121$
h) $4x^2 - 81$

2) Factorise completely:

a) $4x^2 - 5x - 6$
b) $2x^2 - 5x + 3$
c) $15x^2 + 31x + 10$
d) $6x^2 - x - 1$

3) Solve the following equations by first factorising them:

a) $y^2 - 3y = 0$
b) $y^2 - 3y - 4 = 0$
c) $6x^2 - 11x - 7 = 0$
d) $x^2 + 2x = 35$
e) $3y^2 + 5y = 2$

4) Solve the following equations by first completing the square, give answers to 1 dp:

a) $x^2 + 10x + 3 = 0$
b) $x^2 - 8x - 2 = 0$
c) $x^2 + 3x - 4 = 0$

5) Solve the following equations using the quadratic formula, give answers to 1 dp:

a) $x^2 - 4x + 1 = 0$
b) $x^2 - 5x + 1 = 0$
c) $4x^2 + 9x + 1 = 0$
d) $4x^2 - 2x = 3$
e) $1 = x^2 - 8x + 2$

Now check your work, answers on page 18.

Simultaneous Equations

Problems that involve more than one equation and more than one unknown that are to be solved at the same time with the same values are known as *"Simultaneous Equations"*.

Linear Simultaneous Equations, Elimination Method:

Example 1:

Solve the following pair of equations simultaneously:

① $3x + 2y = 29 \rightarrow \times 4$

② $4x - 3y = -1 \rightarrow \times 3$

The aim is to get the same amount of one unknown by multiplying one or both equations by a constant

Unknowns have the SAME SIGN → SUBTRACT
DIFFERENT SIGNS → ADD

③ $12x + 8y = 116$

④ $12x - 9y = -3$ (−)

$17y = 119$

$y = 7$

Substitute into ① to find x: $3x + 14 = 29$

$x = 5$

Check results in ② $4 \times 5 - 3 \times 7 = 20 - 21 = -1$ ✓

Linear Simultaneous Equations, Substitution Method:

Example 2:

Solve the following pair of equations simultaneously:

① $3x + y = 22$

② $5x + 2y = 40$

The aim is to get one of the unknowns on its own and then substitute it into the other equation

① $y = 22 - 3x \rightarrow$ ② $5x + 2(22 - 3x) = 40$

$5x + 44 - 6x = 40$

$x = 4$

Substitute into ① to find y: $y = 22 - 3 \times 4 = 10$

Quadratic Simultaneous Equations:

Example 3:

Solve the following equations simultaneously:

① $y = 2x + 2$
② $y = x^2 - 1$

The aim is to substitute the linear equation into the quadratic equation.

$2x + 2 = x^2 - 1$
$0 = x^2 - 2x - 3$
$0 = (x - 3)(x + 1)$
$x = 3$ or $x = -1$

See Factorising and Solving. Page 4

Substitute into ① to find y:
$x = 3, y = 8$ $x = -1, y = 0$

Exercise 3:

1) Solve the following simultaneous equations using the elimination method:

a) $2x + 5y = 24$
$4x + 3y = 20$

b) $2a + 3b = 9$
$4a + b = 13$

c) $x - 2y = -4$
$3x + y = 9$

d) $5x - 7y = 27$
$3x - 4y = 16$

2) Solve the following simultaneous equations using the substitution method:

a) $x + 3y = 5$
$2x + y = 5$

b) $x - y = 2$
$3x + y = 10$

c) $a + 4b = 6$
$8b - a = -3$

d) $2x = 4 + z$
$6x - 5z = 18$

3) Solve the following linear and quadratic simultaneous equations:

a) $y = x^2 - 2x$
$y = x + 4$

b) $y = 7x - 8$
$y = x^2 - x + 7$

c) $y = x^2 - 3x + 7$
$5x - y = 8$

d) $y = 9x - 4$
$y = 2x^2$

Now check your work, answers on page 18.

Inequalities

Equations involving greater than >, less than <, greater than or equal to ≥ or less than or equal to ≤ are called *"Inequalities"*. Inequalities have a set of solutions.

Linear Inequalities:

REMEMBER
Change the direction of the inequality when you × or ÷ by a negative number.

Example 1:

Solve the following inequalities:

a) $3x + 7 < 31$

$3x < 24$

$x < 8$

b) $8 - 5x \geq 68$

$-5x \geq 60$

$x \leq -12$

See Solving Linear Equations. Page 2

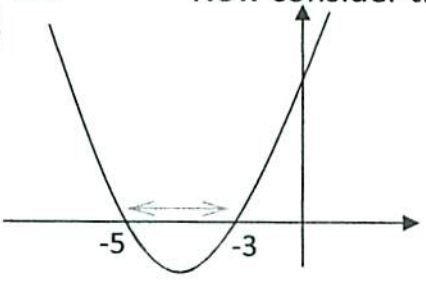
Quadratic Inequalities:

Example 2:

Solve the following inequality: $x^2 + 8x + 15 < 0$

$(x + 3)(x + 5) < 0$ so $x^2 + 8x + 15 = 0$ when $x = -3$ or $x = -5$

Now consider the graph of $y = x^2 + 8x + 15$



So the graph has $y < 0$
when x is between -5 and -3

Solution: $-5 < x < -3$

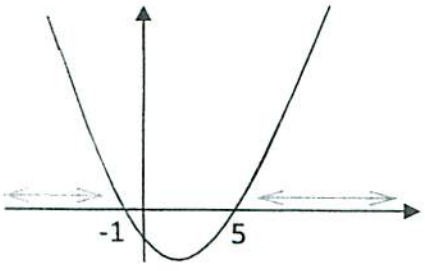
See Solving Quadratic equations. Page 4

Example 3:

Solve the following inequality: $x^2 - 4x - 5 > 0$

$(x - 5)(x + 1) > 0$ so $x^2 - 4x - 5 = 0$ when $x = -1$ or $x = 5$

Now consider the graph of $y = x^2 - 4x - 5$



So the graph has $y > 0$
when x is less than -1 or greater than 5

Solution: $x < -1$ or $x > 5$

(Ensure you write the solutions separately in this case)

Exercise 4:

1) Solve the following inequalities:

a) $x - 3 > 10$
b) $x + 1 < 0$
c) $2x + 1 \leq 6$
d) $5x < x + 1$
e) $3x + 1 < 2x + 5$
f) $2(x + 1) \geq x - 7$
g) $3(x - 1) < 2(1 - x)$
h) $4 - 2x \leq 2$

2) Solve the following quadratic inequalities:

a) $x^2 + 7x + 12 < 0$
b) $x^2 - 8x - 9 > 0$
c) $x^2 - 144 \leq 0$
d) $12x^2 - 16x + 5 < 0$
e) $4x^2 - 3x - 10 > 0$
f) $x^2 - 14x + 49 \leq 0$

Now check your work, answers on page 19.

Algebraic Fractions

Any fraction that involves an unknown is an *"Algebraic Fraction"*. You may be asked to *simplify* expressions or *solve* equations involving algebraic fractions.

Simplifying Algebraic Fractions, Adding & Subtracting:

Example 1:

Simplify the following algebraic fractions:

REMEMBER
Before you can add or subtract fractions you must have the same denominators.

a) $\frac{x}{6}+\frac{x}{8}$

$=\frac{4x}{24}+\frac{3x}{24}=\frac{7x}{24}$

REMEMBER
Only add the numerators.

b) $\frac{3}{x-2}+\frac{5}{x+3}$

$=\frac{3(x+3)}{(x-2)(x+3)}+\frac{5(x-2)}{(x-2)(x+3)}$

$=\frac{3x+9}{(x-2)(x+3)}+\frac{5x-10}{(x-2)(x+3)}$

$=\frac{8x-1}{(x-2)(x+3)}$

c) $\frac{12}{x+5}-\frac{2}{x+1}$

$=\frac{12(x+1)}{(x+5)(x+1)}-\frac{2(x+5)}{(x+1)(x+5)}$

$=\frac{12x+12}{(x+5)(x+1)}-\frac{2x+10}{(x+1)(x+5)}$

$=\frac{12x+12-(2x+10)}{(x+5)(x+1)}$

$=\frac{10x+2}{(x+5)(x+1)}$

Simplifying Algebraic Fractions, Multiplying & Dividing:

Example 2:

Simplify the following algebraic fractions:

REMEMBER
Always factorise before you multiply or divide.

a) $\frac{x}{x^2+5x+6}\times\frac{x^2+3x}{x+1}$

$=\frac{x}{(x+2)(x+3)}\times\frac{x(x+3)}{x+1}$

$=\frac{x^2}{(x+2)(x+1)}$

REMEMBER
Never Divide: Flip the second fraction over & multiply.

b) $\frac{x(x+2)}{x^2+8x-9}\div\frac{x^2+5x}{x-1}$

$=\frac{x(x+2)}{(x+9)(x-1)}\times\frac{x-1}{x(x+5)}$

$=\frac{(x+2)}{(x+9)(x+5)}$

REMEMBER
Cancel whole brackets from any numerator & denominator.

Solving Equations involving Algebraic Fractions:

Example 3:

Solve the following equations:

a) $\frac{x}{x^2+5x+4}\times\frac{x^2+6x+8}{x+2}=3$

$\frac{x}{(x+1)(x+4)}\times\frac{(x+2)(x+4)}{(x+2)}=3$

$\frac{x}{(x+1)}=3$

When you have a single fraction on one side multiply up to remove all fractions.

$x=3(x+1)$

$x=3x+3$

$-2x=3$

$x=-\frac{3}{2}=-1\cdot5$

b) $\frac{3}{x+1}+\frac{5}{x+2}=7$

$\frac{3(x+2)}{(x+1)(x+2)}+\frac{5(x+1)}{(x+1)(x+2)}=7$

$\frac{8x+11}{(x+1)(x+2)}=7$

$8x+11=7(x+1)(x+2)$

$8x+11=7x^2+21x+14$

$7x^2+13x+3=0$ (using quadratic formula)

x = -0.27 or x = -1.59

Exercise 5:

1) Simplify the following algebraic fractions:

a) $\frac{7a^2b}{35ab^2}$

b) $\frac{5ab}{15a+10a^2}$

c) $\frac{18a-3ab}{6a^2}$

d) $\frac{4ab+8a^2}{2ab}$

e) $\frac{x^2+2x}{x^2-3x}$

f) $\frac{x^2-3x}{x^2-2x-3}$

g) $\frac{x^2+6x+5}{x^2-x-2}$

h) $\frac{x^2-4x-21}{x^2-5x-14}$

i) $\frac{x^2+7x+10}{x^2-4}$

2) Write the following expressions as a single fraction:

a) $\frac{x-1}{3}+\frac{x+2}{4}$

b) $\frac{x-3}{3}-\frac{x-2}{5}$

c) $\frac{3}{4x}+\frac{2}{5x}$

d) $\frac{3}{4x}-\frac{2}{3x}$

e) $\frac{3}{x-2}+\frac{4}{x}$

f) $\frac{2}{x+3}-\frac{5}{x-1}$

3) Write the following expressions as a single fraction:

a) $\frac{x^2-3x-40}{x^2+2x}\times\frac{x^2+5x+6}{x^2-25}$

b) $\frac{x^2}{x^2+2x}\div\frac{x}{x+2}$

4) Solve the following equations giving answers to two decimal places where necessary:

a) $\frac{2}{x}+\frac{2}{x+1}=3$

b) $\frac{3}{x-1}+\frac{3}{x+1}=4$

c) $\frac{2}{x-2}+\frac{4}{x+1}=3$

Now check your work, answers on page 19.

Indices

An *"Index"* is also known as a power. The plural of index is *"Indices"*.

Index

Base → 2^3

Simplifying Indices:

You can only simplify indices when the bases are the same.

The Rules:

- $3^2 \times 3^5 = \underbrace{3 \times 3}_{3^2} \times \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{3^5} = 3^7 = 3^{2+5}$ $\quad y^a \times y^b = y^{a+b}$
- $5^6 \div 5^4 = \frac{5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}\,1} = \frac{5^2}{1} = 5^2 = 5^{6-4}$ $\quad y^a \div y^b = y^{a-b}$
- $(3^2)^5 = 3^2 \times 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2+2} = 3^{10} = 3^{2\times5}$ $\quad (y^a)^b = y^{a\times b}$
- $4^0 = 1$ $\quad y^0 = 1$
- $9^{½} = \sqrt{9} = 3, \quad 8^{⅔} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ $\quad y^{n/m} = (\sqrt[m]{y})^n$
- $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ $\quad y^{-n} = \frac{1}{y^n}$

Example 1:

Simplify the following expressions:

a) $c^4 \times c^7$
$= c^{4+7} = c^{11}$

b) $p^4 \div p^{-6}$
$= p^{4-(-6)} = p^{10}$

c) $(r^4)^6$
$= r^{4\times6} = r^{24}$

Example 2:

Evaluate the following:

a) $4^{-½}$
$= \frac{1}{\sqrt{4}} = \frac{1}{2}$

b) $(6^{½})^3 \times 6^{½}$
$= 6^{3/2} \times 6^{½}$
$= 6^{3/2 + ½}$
$= 6^2 = 36$

c) $49^{3/2}$
$= \left(\sqrt{49}\right)^3$
$= 7^3 = 343$

d) $2.25^{-½}$
$= \left(\frac{9}{4}\right)^{-½} = \left(\frac{4}{9}\right)^{½}$
$= \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

Exercise 6:

1) Simplify the following expressions:

a) $x^3 \times x^4$

b) $m^3 \div m^2$

c) $y^{½} \times y^{½}$

d) $w^{-7} \times w^2$

e) $(k^{½})^6$

f) $(x^{-3})^{-2}$

g) $2x^2 \times 3x^2$

h) $(2x)^2 \times (3x)^3$

2) Evaluate the following quantities:

a) $100^{3/2}$

b) $(5^{-4})^{½}$

c) $81^{¼} \div 16^{¼}$

d) $0{\cdot}01^{½}$

e) $0{\cdot}04^{½}$

f) $\left(3\frac{3}{8}\right)^{⅓}$

g) $\left(11\frac{1}{9}\right)^{-½}$

h) $\left(\frac{1}{8}\right)^{-2}$

i) $\left(\frac{9}{25}\right)^{-½}$

Now check your work, answers on page 20.

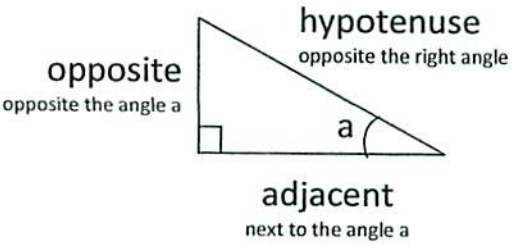
Trigonometry

Trigonometry is the use of Sine, Cosine and Tangent to calculate sides and angles in triangles.

Right-angled Triangles:

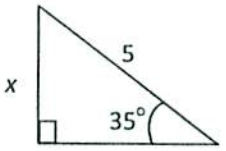
$\text{Sin}(a) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{Cos}(a) = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{Tan}(a) = \frac{\text{opposite}}{\text{adjacent}}$

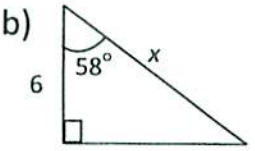


Example 1:

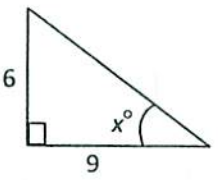
Find the sides and angles indicated: a)



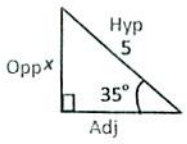
b)



c)



a)

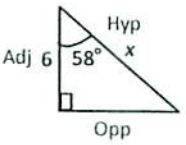


$\sin(35) = \frac{x}{5}$

$5\sin(35) = x$

$x = 2 \cdot 87$

b)

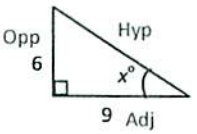


$\cos(58) = \frac{6}{x}$

$x = \frac{6}{\cos(58)}$

$x = 11 \cdot 32$

c)



$\tan(x) = \frac{6}{9}$

$x = \tan^{-1}\left(\frac{6}{9}\right)$

$x = 33 \cdot 7°$

Non right-angled Triangles:

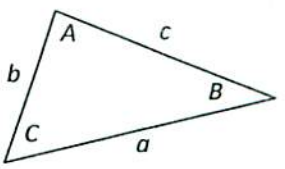
Label angles as capital letters, sides as lower case. The sides are labelled opposite their corresponding angles.

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = c^2 + a^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$



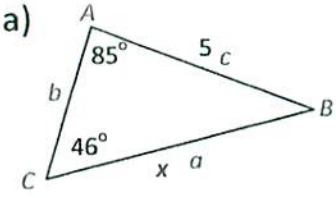
Good Habits:
Write a list of the angles and sides you know.

Which Rule?
If you have a pair use the Sine Rule.

Example 2:

Find the sides and angles indicated:

a)



A = 85 a = x

B = b =

C = 46 c = 5

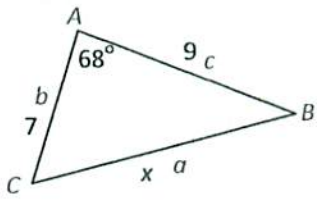
Pair so sine rule:

$$\frac{x}{\sin 85} = \frac{5}{\sin 46}$$

$$x = \frac{5\sin 85}{\sin 46}$$

$$x = 6\cdot 92$$

b)



A = 68 a = x

B = b = 7

C = c = 9

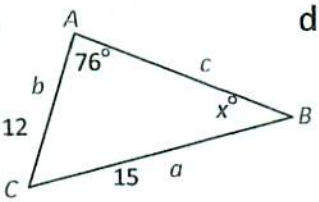
No pair so cosine rule:

$$x^2 = 7^2 + 9^2 - 2\times 7\times 9\times \cos 68$$

$$x^2 = 82\cdot 8$$

$$x = \sqrt{82\cdot 8} = 9\cdot 10$$

c)



A = 76 a = 15

B = x b = 12

C = c =

Pair so sine rule:

$$\frac{15}{\sin 76} = \frac{12}{\sin x}$$

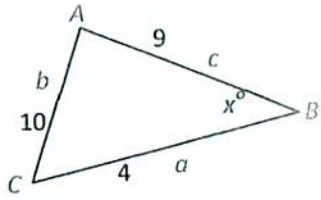
$$\frac{\sin 76}{15} = \frac{\sin x}{12}$$

$$\sin x = \frac{12\sin 76}{15}$$

$$x = \sin^{-1} 0\cdot 78$$

$$x = 50\cdot 9$$

d)



A = a = 4

B = x b = 10

C = c = 9

No pair so cosine rule:

$$10^2 = 4^2 + 9^2 - 2\times 4\times 9\times \cos x$$

$$100 = 16 + 81 - 72\cos x$$

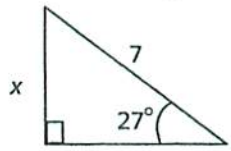
$$\frac{100 - 16 - 81}{-72} = \cos x$$

$$\cos^{-1}\left(-\frac{1}{24}\right) = x = 92\cdot 4^\circ$$

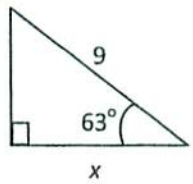
Exercise 7:

1) Find the missing sides and angles:

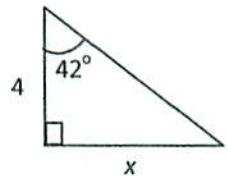
a)



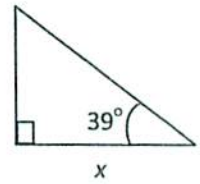
b)



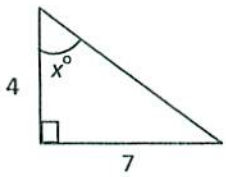
c)



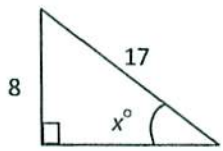
d)



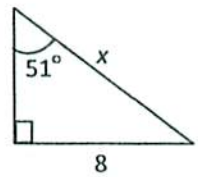
e)



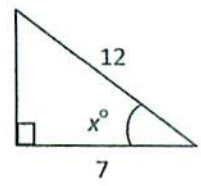
f)



g)

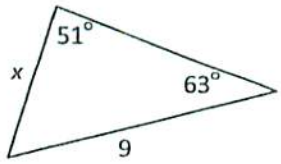


h)

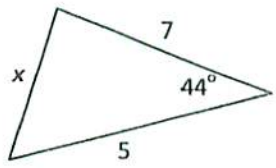


2) Find the missing sides and angles:

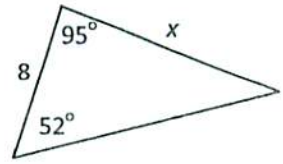
a)



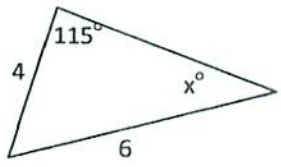
b)



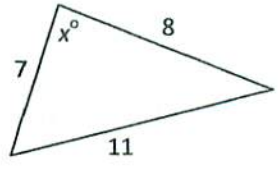
c)



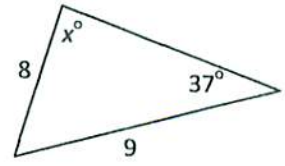
d)



e)



f)



Now check your work, answers on page 20.

Surds

Rules of surds

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{a} \times \sqrt{a} = a$

$a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$

$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$

Simplifying examples

1) $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

2) $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

3) $5\sqrt{3} + 6\sqrt{3} = 11\sqrt{3}$

4) $\sqrt{5} \times \sqrt{5} = 5$

5) $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

↑

Find a square factor

6) $\sqrt{300} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

Rationalising the denominator

1) $\frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

2) $\frac{8}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6} = \frac{4\sqrt{6}}{3}$

3) $\frac{20}{\sqrt{50}} = \frac{20}{\sqrt{25} \times \sqrt{2}} = \frac{20}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{5 \times 2} = \frac{20\sqrt{2}}{10} = 2\sqrt{2}$

EXERCISE 8

(A)

Simplify

1) $\frac{2}{\sqrt{2}}$ 2) $\frac{3}{\sqrt{3}}$ 3) $\frac{4}{\sqrt{4}}$ 4) $\frac{6}{\sqrt{2}}$

5) $\frac{14}{\sqrt{7}}$ 6) $\frac{8}{\sqrt{2}}$ 7) $\frac{9}{\sqrt{3}}$ 8) $\frac{12}{\sqrt{3}}$

9) $\frac{14}{\sqrt{2}}$ 10) $\frac{20}{\sqrt{2}}$ 11) $\frac{30}{\sqrt{3}}$ 12) $\frac{50}{\sqrt{5}}$

13) $\frac{70}{\sqrt{5}}$ 14) $\frac{39}{\sqrt{3}}$ 15) $\frac{49}{\sqrt{7}}$ 16) $\frac{63}{\sqrt{21}}$

(B)

Simplify each of the following by writing as products of whole numbers and surds

1) $\sqrt{8}$ 2) $\sqrt{12}$ 3) $\sqrt{24}$ 4) $\sqrt{28}$

5) $\sqrt{108}$ 6) $\sqrt{40}$ 7) $\sqrt{50}$ 8) $\sqrt{18}$

9) $\sqrt{48}$ 10) $\sqrt{32}$ 11) $\sqrt{20}$ 12) $\sqrt{125}$

13) $\sqrt{200}$ 14) $\sqrt{216}$ 15) $\sqrt{192}$ 16) $\sqrt{320}$

(C)

Simplify

1) $\sqrt{2}+2\sqrt{2}$ 2) $\sqrt{3}+3\sqrt{3}$ 3) $2\sqrt{2}+3\sqrt{2}$

4) $\sqrt{8}+\sqrt{2}$ 5) $\sqrt{8}-\sqrt{2}$ 6) $\sqrt{12}-\sqrt{3}$

7) $2\sqrt{5}-\sqrt{5}$ 8) $\sqrt{32}-2\sqrt{2}$ 9) $2\sqrt{5}-\sqrt{5}$

10) $3\sqrt{5}-2\sqrt{5}$ 11) $4\sqrt{7}-\sqrt{28}$ 12) $\sqrt{500}-3\sqrt{5}$

13 $\sqrt{12}+3\sqrt{75}$ 14 $\sqrt{200}+\sqrt{18}-2\sqrt{72}$

15 $\sqrt{20}+2\sqrt{45}-3\sqrt{80}$ 16 $5\sqrt{6}-\sqrt{24}+\sqrt{294}$

17 $\sqrt{63}-2\sqrt{28}+\sqrt{175}$

Rationalise the denominators:

18 $\frac{1}{\sqrt{2}}$ 19 $\frac{1}{\sqrt{7}}$ 20 $\frac{7}{\sqrt{5}}$

21 $\frac{\sqrt{2}}{3\sqrt{3}}$ 22 $\frac{\sqrt{8}}{\sqrt{32}}$ 23 $\frac{\sqrt{5}}{\sqrt{45}}$

Answers

Exercise 1:

1)
a) $q = 7$
b) $p = 7$
c) $y = \frac{1}{8}$
d) $n = 12$
e) $b = 6$
f) $c = 4$

2)
a) $x = 18$
b) $x = 28$
c) $x = 45$
d) $x = \frac{3}{10}$

3)
a) $m = 3$
b) $p = -7$
c) $q = 1\frac{1}{6}$
d) $x = 2$

4)
a) $e = -1\frac{1}{2}$
b) $f = 1$
c) $g = 2\frac{3}{4}$
d) $h = 18$

5)
a) $x = 3$
b) $y = \frac{1}{5}$
c) $x = 2$
d) $x = 3$
e) $x = \frac{3}{5}$
f) $x = -4$

Exercise 2:

1)
a) $(x - 1)(x - 4)$
b) $(x + 5)(x + 2)$
c) $(x - 5)(x + 3)$
d) $(x + 2)(x + 3)$
e) $(x - 1)(x - 10)$
f) $(x + 3)(x - 4)$
g) $(x - 11)(x + 11)$
h) $(2x - 9)(2x + 9)$

2)
a) $(4x + 3)(x - 2)$
b) $(2x - 1)(x - 1)$
c) $(5x + 2)(3x + 5)$
d) $(3x + 1)(2x - 1)$

3)
a) $y(y - 3) = 0$
$y = 0$ or $y = 3$
b) $(y - 4)(y + 1) = 0$
$y = 4$ or $y = -1$
c) $(2x + 1)(3x - 7) = 0$
$x = -\frac{1}{2}$ or $x = 2\frac{1}{3}$
d) $(x + 7)(x - 5) = 0$
$x = -7$ or $x = 5$
e) $(3y - 1)(y + 2) = 0$
$y = \frac{1}{3}$ or $y = -2$

4)
a) $(x + 5)^2 - 22 = 0$
$x = -0{\cdot}3$ or $x = -9{\cdot}7$
b) $(x - 4)^2 - 18 = 0$
$x = -0{\cdot}2$ or $x = 8{\cdot}2$
c) $(x + \frac{3}{2})^2 - \frac{25}{4} = 0$
$x = 1$ or $x = -4$

5)
a) $x = 3{\cdot}7$ or $x = 0{\cdot}3$
b) $x = 4{\cdot}8$ or $x = 0{\cdot}2$
c) $x = -0{\cdot}1$ or $x = -2{\cdot}1$
d) $x = 1{\cdot}2$ or $x = -0{\cdot}7$
e) $x = 7{\cdot}9$ or $x = 0{\cdot}1$

Exercise 3:

1)
a) $x = 2, y = 4$
b) $a = 3, b = 1$
c) $x = 2, y = 3$
d) $x = 4, y = -1$

2)
a) $x = 2, y = 1$
b) $x = 3, y = 1$
c) $a = 5, b = \frac{1}{4}$
d) $x = \frac{1}{2}, z = -3$

3)
a) $x = 4, y = 8$
$x = -1, y = 3$
b) $x = 3, y = 13$
$x = 5, y = 27$
c) $x = 3, y = 7$
$x = 5, y = 17$
d) $x = \frac{1}{2}, y = \frac{1}{2}$
$x = 4, y = 32$

Exercise 4:

1)

a) $x > 13$
b) $x < -1$
c) $x \leq 2{\cdot}5$
d) $x < ¼$
e) $x < 4$
f) $x \geq -9$
g) $x < 1$
h) $x \geq 1$

2)

a) $-4 < x < -3$
b) $x < -1, x > 9$
c) $-12 \leq x \leq 12$
d) $½ < x < ⅚$
e) $x < -1¼, x > 2$
f) $x = 7$

Exercise 5:

1)

a) $\frac{a}{5b}$
b) $\frac{b}{3+2a}$
c) $\frac{6-b}{2a}$
d) $\frac{2(b+2a)}{b}$
e) $\frac{x+2}{x-3}$
f) $\frac{x}{x+1}$
g) $\frac{x+5}{x-2}$
h) $\frac{x+3}{x+2}$
i) $\frac{x+5}{x-2}$

2)

a) $\frac{7x+2}{12}$
b) $\frac{2x-9}{15}$
c) $\frac{23}{20x}$
d) $\frac{1}{12x}$
e) $\frac{7x-8}{x(x-2)}$
f) $\frac{-3x-17}{(x+3)(x-1)}$

3)

a) $\frac{(x-8)(x+3)}{x(x-5)}$
b) 1

4)

a) $x = -⅔$ or $x = 1$
b) $x = -½$ or $x = 2$
c) $x = 0$ or $x = 3$

Exercise 6:

1)

a) x^7
b) m
c) y
d) w^{-5}
e) k^3
f) x^6
g) $6x^4$
h) $108x^5$

2)

a) 1000
b) $\frac{1}{25}$
c) 1·5
d) $\frac{1}{10}$
e) $\frac{1}{5}$
f) 1·5
g) $\frac{3}{10}$
h) 64
i) $1\frac{2}{3}$

Exercise 7:

1)

a) $x = 3{\cdot}18$
b) $x = 4{\cdot}09$
c) $x = 3{\cdot}60$
d) $x = 6{\cdot}17$
e) $x = 60{\cdot}3^\circ$
f) $x = 28{\cdot}1^\circ$
g) $x = 10{\cdot}29$
h) $x = 54{\cdot}3^\circ$

2)

a) $x = 10{\cdot}32$
b) $x = 4{\cdot}86$
c) $x = 11{\cdot}57$
d) $x = 37{\cdot}2^\circ$
e) $x = 94{\cdot}1^\circ$
f) $x = 42{\cdot}6^\circ$

EXERCISE 8

(A)

1) $\sqrt{2}$	2) $\sqrt{3}$	3) 2	4) $3\sqrt{2}$
5) $2\sqrt{7}$	6) $4\sqrt{2}$	7) $3\sqrt{3}$	8) $4\sqrt{3}$
9) $7\sqrt{2}$	10) $10\sqrt{2}$	11) $10\sqrt{3}$	12) $10\sqrt{5}$
13) $14\sqrt{5}$	14) $13\sqrt{3}$	15) $7\sqrt{7}$	16) $3\sqrt{21}$

(B)

1) $2\sqrt{2}$	2) $2\sqrt{3}$	3) $2\sqrt{6}$	4) $2\sqrt{7}$
5) $6\sqrt{3}$	6) $2\sqrt{10}$	7) $5\sqrt{2}$	8) $3\sqrt{2}$
9) $4\sqrt{3}$	10) $4\sqrt{2}$	11) $2\sqrt{5}$	12) $5\sqrt{5}$
13) $10\sqrt{2}$	14) $6\sqrt{6}$	15) $8\sqrt{3}$	16) $8\sqrt{5}$

(C)

1) $3\sqrt{2}$	2) $4\sqrt{3}$	3) $5\sqrt{2}$	4) $3\sqrt{2}$
5) $\sqrt{2}$	6) $\sqrt{3}$	7) $\sqrt{5}$	8) $2\sqrt{2}$
9) $\sqrt{5}$	10) $\sqrt{5}$	11) $2\sqrt{7}$	12) $7\sqrt{5}$

13 $17\sqrt{3}$	**14** $\sqrt{2}$	**15** $-4\sqrt{5}$	**16** $10\sqrt{6}$
17 $4\sqrt{7}$	**18** $\frac{\sqrt{2}}{2}$	**19** $\frac{\sqrt{7}}{7}$	**20** $\frac{7\sqrt{5}}{5}$
21 $\frac{\sqrt{6}}{9}$	**22** $\frac{1}{2}$	**23** $\frac{1}{3}$	**24** $\frac{\sqrt{7}}{7}$