AQA Level 2 Further mathematics Coordinate geometry

Section 1: Points and straight lines

Solutions to Exercise

1. (a) (i) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-4}{3-7} = \frac{-3}{-4} = \frac{3}{4}$.

(ii) Gradient m of perpendicular line satisfies $m \times \frac{3}{4} = -1$

so gradient of perpendicular line $= -\frac{4}{3}$.

(iii) Midpoint of AB $= \left(\frac{3+7}{2}, \frac{1+4}{2}\right) = (5, 2.5)$

(iv) Distance AB $= \sqrt{(3-7)^2 + (1-4)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

(v) Distance from A to B in x-direction = 7 – 3 = 4

x-coordinate of C $= 3 + \frac{3}{5} \times 4 = 3 + 2.4 = 5.4$

Distance from A to B in y-direction = 4 – 1 = 3

y-coordinate of C $= 1 + \frac{3}{5} \times 3 = 1 + 1.8 = 2.8$

C = (5.4, 2.8)

(b) (i) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{9-(-1)}{-2-3} = \frac{10}{-5} = -2$.

(ii) Gradient m of perpendicular line satisfies $m \times -2 = -1$

so gradient of perpendicular line $= \frac{1}{2}$.

(iii) Midpoint of AB $= \left(\frac{-2+3}{2}, \frac{9+(-1)}{2}\right) = (0.5, 4)$

(iv) Distance AB $= \sqrt{(-2-3)^2 + (9-(-1))^2}$

$= \sqrt{25+100}$

$= \sqrt{125}$

$= 5\sqrt{5}$

(v) Distance from A to B in x-direction = 3 – (-2) = 5

x-coordinate of C $= -2 + \frac{3}{5} \times 5 = -2 + 3 = 1$

AQA FM Coordinate geometry 1 Exercise solutions

Distance from A to B in y-direction = -1 – 9 = -10
y-coordinate of C $= 9 + \frac{3}{5} \times -10 = 9 - 6 = 3$
C = (1, 3)

2. (i) $M = \left(\dfrac{2+(-4)}{2}, \dfrac{-1+8}{2} \right) = (-1, 3.5)$

(ii) Distance from P to Q in x-direction = -4 – 2 = -6
x-coordinate of R $= 2 + \frac{1}{4} \times -6 = 2 - 1.5 = 0.5$
Distance from P to Q in y-direction = 8 – (-1) = 9
y-coordinate of R $= -1 + \frac{1}{4} \times 9 = -1 + 2.25 = 1.25$
C = (0.5, 1.25)

(iii) Distance from P to Q in x-direction = -4 – 2 = -6
x-coordinate of S $= 2 + \frac{7}{10} \times -6 = 2 - 4.2 = -2.2$
Distance from P to Q in y-direction = 8 – (-1) = 9
y-coordinate of S $= -1 + \frac{7}{10} \times 9 = -1 + 6.3 = 5.3$
C = (-2.2, 5.3)

3. (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-y}{3-6} = \dfrac{1-y}{-3}$

Gradient of AB = 2 $\Rightarrow \dfrac{1-y}{-3} = 2$

$\Rightarrow 1 - y = -6$

$y = 7$

(ii) Distance AB is 5

$\sqrt{(3-6)^2 + (1-y)^2} = 5$

$9 + (1-y)^2 = 25$

$(1-y)^2 = 16$

$1 - y = \pm 4$

$y = 1 - 4$ or $1 + 4$

$y = -3$ or 5

(iii) If A, B and C are collinear, gradient of AB = gradient of AC.

Gradient of AC $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-(-2)}{3-12} = \dfrac{3}{-9} = -\dfrac{1}{3}$

From (i), gradient of AB $= \dfrac{1-y}{-3}$

$$\frac{1-y}{-3} = -\frac{1}{3}$$
$$1-y=1$$
$$y=0$$

(iv) If AB is perpendicular to BC, then grad AB × grad BC = -1

From (i), gradient of AB $= \dfrac{1-y}{-3}$

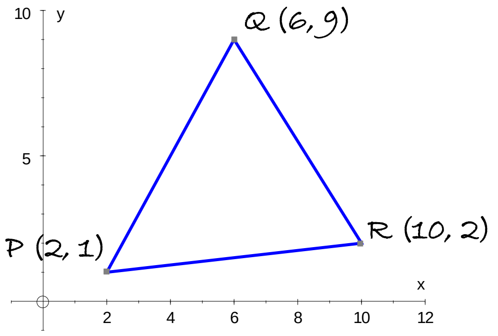
Gradient of BC $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y-(-2)}{6-12} = \dfrac{y+2}{-6}$

$$\frac{1-y}{-3} \times \frac{y+2}{-6} = -1$$
$$(1-y)(y+2) = -18$$
$$2 - y - y^2 = -18$$
$$y^2 + y - 20 = 0$$
$$(y+5)(y-4) = 0$$
$$y = -5 \text{ or } y = 4$$

(v) Length AB = length BC

$$\sqrt{(3-6)^2 + (1-y)^2} = \sqrt{(6-12)^2 + (y-(-2))^2}$$
$$9 + (1-y)^2 = 36 + (y+2)^2$$
$$1 - 2y + y^2 = 27 + y^2 + 4y + 4$$
$$0 = 6y + 30$$
$$y = -5$$

4. (i)



(ii) PQ $= \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

PR $= \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$

QR $= \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ.

Let M be the midpoint of PQ

$M = \left(\frac{2+6}{2}, \frac{1+9}{2}\right) = (4, 5)$

Height of triangle is $MR = \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$

$$\begin{aligned}\text{Area of triangle} &= \tfrac{1}{2} \times PQ \times MR \\ &= \tfrac{1}{2}\sqrt{80}\sqrt{45} \\ &= \tfrac{1}{2}\sqrt{16\times 5}\sqrt{9\times 5} \\ &= \tfrac{1}{2}\times 4\sqrt{5}\times 3\sqrt{5} \\ &= 6\times 5 \\ &= 30\end{aligned}$$

5. Gradient of $EF = \frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$

Gradient of $FG = \frac{5-3}{3-1} = \frac{2}{2} = 1$

Gradient of $GH = \frac{1-5}{4-3} = \frac{-4}{1} = -4$

Gradient of $EH = \frac{1-(-1)}{4-2} = \frac{2}{2} = 1$

EF is parallel to GH and FG is parallel to EH
so EFGH is a parallelogram.

6. (i) Gradient of $y = 4x - 1$ is 4
Gradient of parallel line = 4
Equation of line is $y - 3 = 4(x-2)$

$$\begin{aligned} y - 3 &= 4x - 8 \\ y &= 4x - 5 \end{aligned}$$

(ii) Gradient of $y = 2x + 7$ is 2
Gradient of perpendicular line is $-\frac{1}{2}$
Equation of line is $y - 2 = -\frac{1}{2}(x-1)$

$$\begin{aligned} 2(y-2) &= -(x-1) \\ 2y - 4 &= -x + 1 \\ 2y + x &= 5 \end{aligned}$$

(iii) $3y + x = 10 \Rightarrow y = -\frac{1}{3}x + \frac{10}{3}$
Gradient is $-\frac{1}{3}$
Gradient of parallel line is $-\frac{1}{3}$

Equation of line is $y-(-1)=-\frac{1}{3}(x-4)$

$$3(y+1)=-(x-4)$$
$$3y+3=-x+4$$
$$3y+x=1$$

(iv) $3x+4y=12 \Rightarrow y=-\frac{3}{4}x+3$

Gradient is $-\frac{3}{4}$

Gradient of perpendicular line is $\frac{4}{3}$

Equation of line is $y-0=\frac{4}{3}(x-(-3))$

$$3y=4(x+3)$$
$$3y=4x+12$$

(v) $x+5y+8=0 \Rightarrow y=-\frac{1}{5}x-\frac{8}{5}$

Gradient is $-\frac{1}{5}$

Gradient of parallel line is $-\frac{1}{5}$

Equation of line is $y-(-6)=-\frac{1}{5}(x-(-1))$

$$5(y+6)=-(x+1)$$
$$5y+30=-x-1$$
$$5y+x+31=0$$

7. (i) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{6-2}{1-3}=\dfrac{4}{-2}=-2$

Gradient of perpendicular bisector $=\frac{1}{2}$

Midpoint of AB $=\left(\dfrac{1+3}{2},\dfrac{6+2}{2}\right)=(2,4)$

Gradient of perpendicular bisector is $y-4=\frac{1}{2}(x-2)$

$$2y-8=x-2$$
$$2y=x+6$$

(ii) Gradient of AB $=\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{-1-3}{8-(-2)}=\dfrac{-4}{10}=-\dfrac{2}{5}$

Gradient of perpendicular bisector $=\frac{5}{2}$

Midpoint of AB $=\left(\dfrac{8+-2}{2},\dfrac{-1+3}{2}\right)=(3,1)$

Gradient of perpendicular bisector is $y-1=\frac{5}{2}(x-3)$

$$2y-2=5x-15$$
$$2y=5x-13$$

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(iii) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{2-(-4)}{-5-7} = \dfrac{6}{-12} = -\dfrac{1}{2}$

Gradient of perpendicular bisector $= 2$

Midpoint of AB $= \left(\dfrac{-5+7}{2}, \dfrac{2+-4}{2}\right) = (1,-1)$

Gradient of perpendicular bisector is $y-(-1) = 2(x-1)$

$$y+2 = 2x-2$$

$$y = 2x-4$$

(iv) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-5-1}{-3-5} = \dfrac{-6}{-8} = \dfrac{3}{4}$

Gradient of perpendicular bisector $= -\frac{4}{3}$

Midpoint of AB $= \left(\dfrac{-3+5}{2}, \dfrac{-5+1}{2}\right) = (1,-2)$

Gradient of perpendicular bisector is $y-(-2) = -\frac{4}{3}(x-1)$

$$3y+6 = -4x+4$$

$$3y+4x+2 = 0$$

8. (i) Midpoint of EF $= \left(\dfrac{2+4}{2}, \dfrac{5+1}{2}\right) = (3,3)$

Midpoint of FG $= \left(\dfrac{4+(-2)}{2}, \dfrac{1+(-3)}{2}\right) = (1,-1)$

Midpoint of EG $= \left(\dfrac{2+(-2)}{2}, \dfrac{5+(-3)}{2}\right) = (0,1)$

Median from midpoint of EF (3, 3) to G (-2, -3)

Gradient of median $= \dfrac{-3-3}{-2-3} = \dfrac{-6}{-5} = \dfrac{6}{5}$

Equation of median is $y-3 = \frac{6}{5}(x-3)$

$$5(y-3) = 6(x-3)$$

$$5y-15 = 6x-18$$

$$5y = 6x-3$$

Median from midpoint of FG (1, -1) to E (2, 5)

Gradient of median $= \dfrac{5-(-1)}{2-1} = \dfrac{6}{1} = 6$

Equation of median is $y-(-1) = 6(x-1)$

$$y+1 = 6x-6$$

$$y = 6x-7$$

C (2, 5)

B (9, 4)

Median from midpoint of EG (0, 1) to F (4, 1)

Gradient of median $= \dfrac{1-1}{4-0} = \dfrac{0}{4} = 0$

A (3, 2)

Equation of median is $y = 1$

(ii) Equation of first median is $5y = 6x - 3$

Substituting $x = \frac{4}{3}$ gives $5y = 6 \times \frac{4}{3} - 3 = 8 - 3 = 5$

$y = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of second median is $y = 6x - 7$

Substituting $x = \frac{4}{3}$ gives $y = 6 \times \frac{4}{3} - 7 = 8 - 7 = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of third median is $y = 1$, so $\left(\frac{4}{3}, 1\right)$ lies on the median.

9. (i) B is the intersection point of $y = 4x - 3$ and $y = 2x + 1$

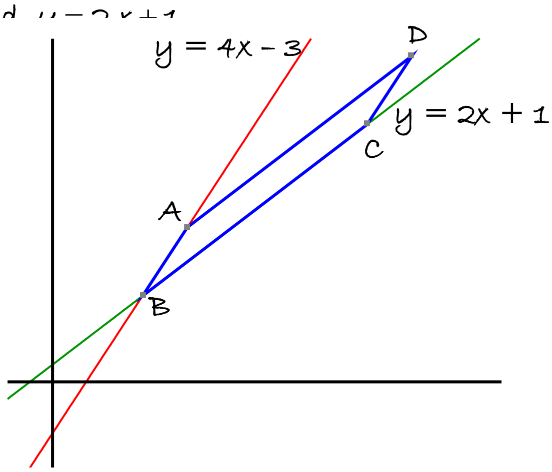
$4x - 3 = 2x + 1$

$2x = 4$

$x = 2$

When $x = 2$, $y = 4 \times 2 - 3 = 5$

The coordinates of B are (2, 5).



(ii) AD is parallel to BC, so AD has gradient 2.

AD passes through the point (3, 9).

Equation of AD is $y - 9 = 2(x - 3)$

$y - 9 = 2x - 6$

$y = 2x + 3$

(iii) CD is parallel to AB, so CD has gradient 4.

CD passes through the point (7, 15).

Equation of CD is $y - 15 = 4(x - 7)$

$y - 15 = 4x - 28$

$y = 4x - 13$

(iv) D is the intersection point of AD and CD.

$2x + 3 = 4x - 13$

$16 = 2x$

$x = 8$

When $x = 8$, $y = 2 \times 8 + 3 = 19$

The coordinates of D are (8, 19).

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10. The diagonals of the rhombus cross at the midpoint of OB, ie at (3, 2).

OB has gradient $\frac{2}{3}$.

AD is perpendicular to OB so has gradient -1.5.

The equation of AD is $(y-2) = -1.5(x-3)$

$$y = -1.5x + 6.5$$

D is on the x-axis so at D: $0 = -1.5x + 6.5$

$$x = \frac{6.5}{1.5} = 4\frac{1}{3}$$

AB = OD and AB is parallel to the x-axis so A is the point $\left(1\frac{2}{3},\ \ 4\right)$.

11. The gradient of AB is $-\frac{5}{2}$.

AD is perpendicular to AB and so has gradient $\frac{2}{5}$.

Point D has coordinates (7, 2).

The coordinates of E are $\left(\frac{2\times2+7}{3},\ \ \frac{2\times0+2}{3}\right) = \left(\frac{11}{3},\ \ \frac{2}{3}\right)$

The gradient of BE is $-\frac{13/3}{11/3} = -\frac{13}{11}$

BE is the line $y = -\frac{13}{11}x + 5$. This crosses the x-axis at $\left(\frac{55}{13},\ \ 0\right)$.