AQA Level 2 Further mathematics Coordinate geometry

Section 1: Points and straight lines

Notes and Examples

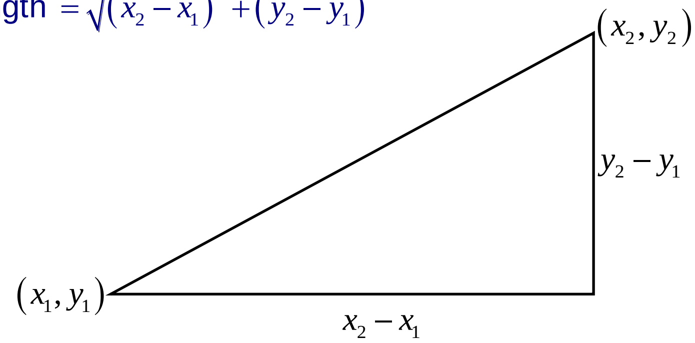
These notes contain sub-sections on:

- Distance between two points
- Midpoints and other points of intersection
- Parallel and perpendicular lines

Distance between two points

The length of a line joining two points (x_1, y_1) and (x_2, y_2) can be found using Pythagoras' Theorem.

Length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example 1

A is the point (2, -6). B is the point (-3, 4). Find the length of AB.

Solution

The distance AB is given by

$$
\begin{aligned}
d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
&= \sqrt{(2-(-3))^2 + ((-6)-4)^2} \\
&= \sqrt{(5)^2 + (-10)^2} \\
&= \sqrt{25+100} \\
&= \sqrt{125}
\end{aligned}
$$

Note: The answer is often left like this if the square root is not exact. However since 125 = 25×5 then $\sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$ is perhaps a simpler form.

To see more examples like these, try the Flash resource ***Distance between two points***.

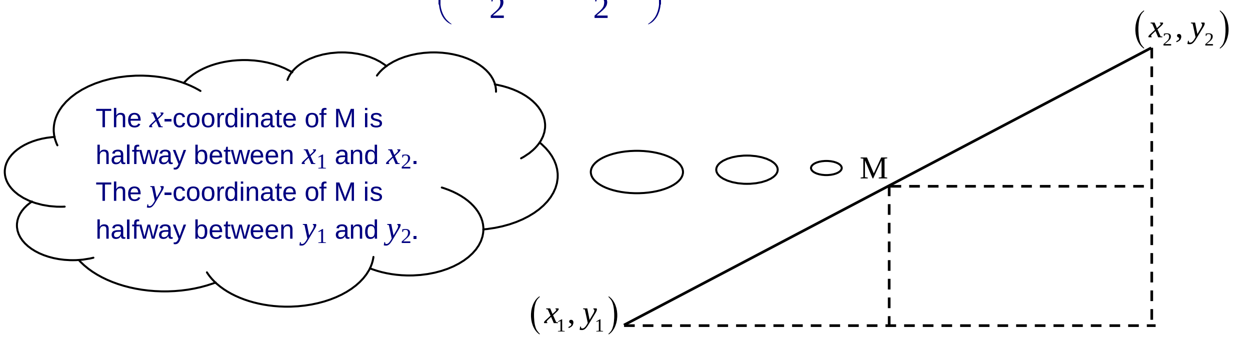
For further practice in examples like the one above, try the interactive questions ***The distance between two points***.

AQA FM Coordinate geometry 1 Notes & Examples

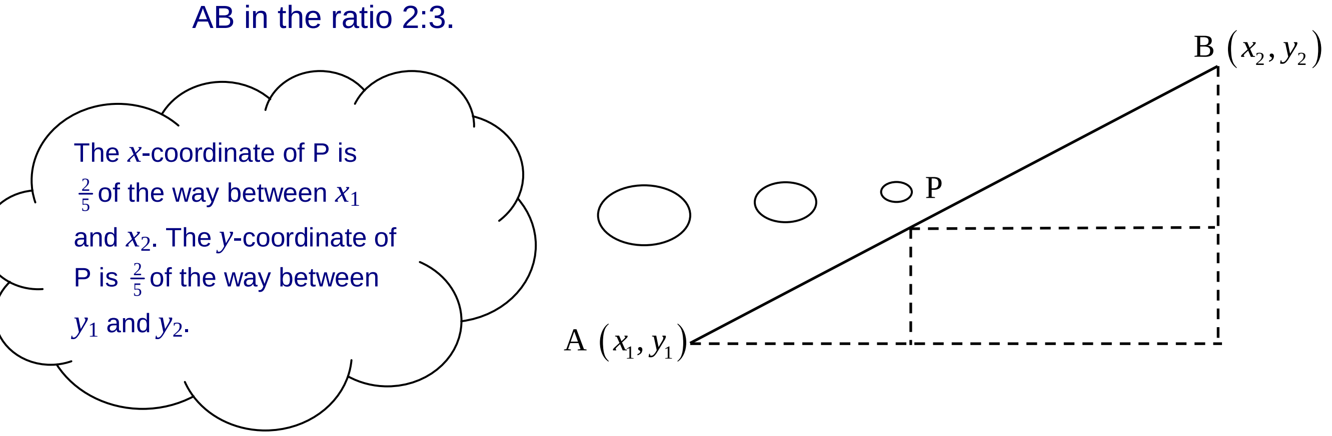
Midpoints and other points of intersection

The midpoint of a line joining two points (x_1, y_1) and (x_2, y_2) is given by

Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$



Similarly, you can find the coordinates of any point on a line which divides the line in a given ratio. For example, the diagram below shows a point P which divides the line AB in the ratio 2:3.



So the coordinates of P are $\left(x_1 + \frac{2}{5}(x_2 - x_1), y_1 + \frac{2}{5}(y_2 - y_1)\right)$

Example 2
A is the point (2, -6). B is the point (-3, 4).
(i) Find the midpoint of AB
(ii) The point C divides the line AB in the ratio 3:1. Find the coordinates of C.

Choose A as (x_1, y_1) and B as (x_2, y_2).

or vice versa, it will still give the same answer (***WHY***?)

Solution

(i) Midpoint is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

i.e. $\left(\dfrac{2 + (-3)}{2}, \dfrac{-6 + 4}{2} \right)$

$= \left(\dfrac{-1}{2}, -1 \right)$

(ii) The distance between A and B in the x-direction is -5.
The x-coordinate of C is $2+\frac{3}{4}\times -5 = 2-3.75 = -1.75$
The distance between A and B in the y-direction is 10.
The y-coordinate of C is $-6+\frac{3}{4}\times 10 = -6+7.5 = 1.5$
C is the point (-1.75, 1.5).

To see more examples like these, try the Flash resource ***Midpoint of two points***. You may also find the Mathcentre video ***Properties of straight line segments*** useful.

For further practice in examples like the one above, try the interactive questions ***The midpoint between two points***.

Parallel and perpendicular lines

If two lines are parallel, they have the same gradient.
If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 m_2 = -1$

Example 3
P is the point (-3, 7). Q is the point (5, 1).
Calculate
(i) the gradient of PQ
(ii) the gradient of a line parallel to PQ
(iii) the gradient of a line perpendicular to PQ.

Solution
(i) Choose P as (x_1, y_1) and Q as (x_2, y_2).

or vice versa: it will still give the same answer (***WHY***?)

Gradient of PQ = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-7}{5-(-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$

Notes:
(1) Draw a sketch and check that your answer is sensible (e.g. has negative gradient).
(2) Check that you get the same result when you choose Q as (x_1, y_1) and P as (x_2, y_2).

(ii) When two lines are parallel their gradients are equal. $(m_1 = m_2)$
So the gradient of the line parallel to PQ is also $-\dfrac{3}{4}$.

(iii) When two lines are perpendicular $m_1 m_2 = -1$.

So $-\frac{3}{4}m_2 = -1$

$\Rightarrow m_2 = \frac{4}{3}$

The gradient of a line perpendicular to PQ is $\frac{4}{3}$.

For further practice in examples like the one above, try the interactive questions ***The gradient of a perpendicular***.

Example 4

A straight line L has equation $y = 2x - 5$.

(i) Find the equation of the line parallel to L and passing through (3, -1).

(ii) Find the equation of the line perpendicular to L and passing through (3, -1).

Solution

Line L has gradient 2.

(i) Any line parallel to L has gradient 2.

The equation of the line is $y - y_1 = m(x - x_1)$

$m = 2$ and (x_1, y_1) is (3, -1)

$\Rightarrow y - (-1) = 2(x - 3)$

$\Rightarrow y + 1 = 2x - 6$

$\Rightarrow y = 2x - 7$

You should check that the point (3, -1) satisfies your line. If it doesn't, you must have made a mistake!

(ii) For two perpendicular lines $m_1 m_2 = -1$, so the gradient of the new line is $-\frac{1}{2}$.

The equation of the line is $y - y_1 = m(x - x_1)$

$m = -\frac{1}{2}$ and (x_1, y_1) is (3, -1)

$\Rightarrow y - (-1) = -\frac{1}{2}(x - 3)$

$\Rightarrow -2y - 2 = x - 3$

$\Rightarrow -2y = x - 1$

$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2}$

The final form of the equation can be written in various different ways:

e.g. $2y = -x + 1$ (This form has no fractions.)

e.g. $2y + x = 1$ (This has no fractions and avoids having a negative sign at the start of the right hand side.)

You can see more examples like ... lines and ***Perpendicular lines***.

The **perpendicular bisector** of a line joining two points A and B is the line that is perpendicular to AB and passes through the midpoint of AB (i.e. it **bisects** AB).

Example 5
Find the equation of the perpendicular bisector of the points A (3, 2) and B (-1, 8).

Solution

Gradient of AB $= \dfrac{8-2}{-1-3} = \dfrac{6}{-4} = -\dfrac{3}{2}$

Gradient of perpendicular line $= \dfrac{2}{3}$

Midpoint of AB is $\left(\dfrac{3+-1}{2}, \dfrac{2+8}{2}\right) = (1,5)$

The equation of the line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 5 = \tfrac{2}{3}(x-1)$$
$$\Rightarrow 3y - 15 = 2x - 2$$
$$\Rightarrow 3y = 2x + 13$$