AQA Level 2 Further mathematics Further algebra

Section 4: Proof and sequences

Exercise

1. n is a positive integer. Prove that $n^3 - n^2$ is always even.

2. Prove that the product of three consecutive integers is always be a multiple of 6.

3. Prove that the square of any odd number is always 1 more than a multiple of 8.

4. Write down the first four terms of each sequence defined below, starting with $n = 1$ in each case.
   (i) $3n-1$
   (ii) n^2-1
   (iii) $3n^2-2n+1$

5. Find a formula for the nth term of each the linear sequences below.
   (i) 2, 5, 8, 11, ...
   (ii) 10, 8, 6, 4, ...

6. Find a formula for the nth term of each the quadratic sequences below.
   (i) 3, 9, 17, 27, 39, ...
   (ii) -2, 4, 14, 28, 46, …
   (iii) 7, 12, 15, 16, 15, …

7. For each of the following sequences, find the 1st term, the 5th term, the 100th term, and the limit of the sequence as $n \to \infty$.
   (i) nth term $= \dfrac{2n+5}{4n-1}$
   (ii) nth term $= \dfrac{1-6n}{2n+3}$

8. The nth term of a sequence is given by the formula n^2+2n-5.
   Prove that 1000 cannot be a term of the sequence.

9. A sequence has all its terms positive.
   As $n \to \infty$, the nth term of the sequence approaches 3.

   (a) Give a possible formula for the nth term of an increasing sequence with the properties above.

   (b) Give a possible formula for the nth term of a decreasing sequence with the properties above.