AQA Level 2 Further mathematics Further algebra

Section 3: Inequalities and indices

Solutions to Exercise

1. (i) $2x + 3 < 10$

$2x < 7$

$x < \frac{7}{2}$

(ii) $5x + 3 \geq 2x - 9$

$3x + 3 \geq -9$

$3x \geq -12$

$x \geq -4$

(iii) $4x + 1 \leq 6x - 7$

$1 \leq 2x - 7$

$8 \leq 2x$

$4 \leq x$

$x \geq 4$

(iv) $5(x - 3) \leq 2(2x + 3)$

$5x - 15 \leq 4x + 6$

$x - 15 \leq 6$

$x \leq 21$

(v) $4(2x + 5) \geq 3(3x - 1)$

$8x + 20 \geq 9x - 3$

$20 \geq x - 3$

$23 \geq x$

$x \leq 23$

(vi) $\frac{2x + 1}{3} > \frac{x - 4}{2}$

$2(2x + 1) > 3(x - 4)$

$4x + 2 > 3x - 12$

$x + 2 > -12$

$x > -14$

AQA FM Further algebra 3 Exercise solutions

2. (i) $3x-1>7-x$

$4x-1>7$

$4x>8$

$x>2$

The smallest integer value that satisfies the inequality is 3.

(ii) $2(1-x)>3x+4$

$2-2x>3x+4$

$2>5x+4$

$-2>5x$

$-\frac{2}{5}>x$

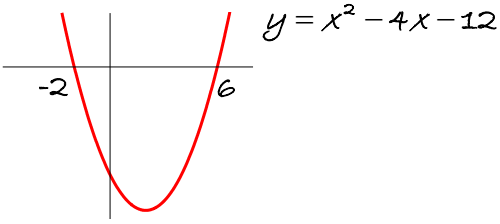
$x<-\frac{2}{5}$

The largest integer value that satisfies the inequality is -1.

3. (i) $x^2-4x-12\leq 0$

$(x-6)(x+2)\leq 0$

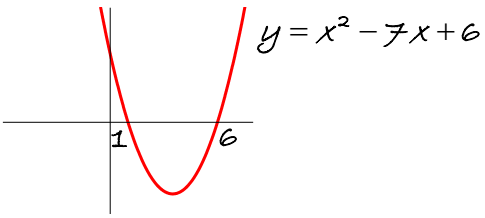
From graph, $-2\leq x\leq 6$



(ii) $x^2-7x+6>0$

$(x-1)(x-6)>0$

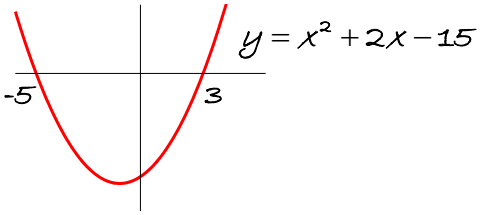
From graph, $x<1$ or $x>6$



(iii) $x^2+2x-15\geq 0$

$(x+5)(x-3)\geq 0$

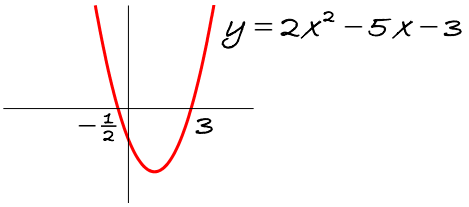
From graph, $x\leq -5$ or $x\geq 3$



(v) $3x^2+5x+2<0$

$(3x+2)(x+1)<0$

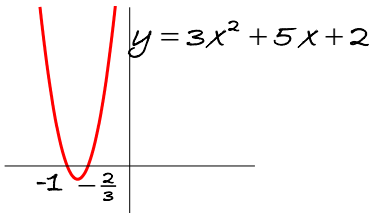
From graph, $-1<x<-\frac{2}{3}$



(vi) $4x^2-4x-3>0$

$(2x-3)(2x+1)>0$

From graph, $x<-\frac{1}{2}$ or $x>\frac{3}{2}$



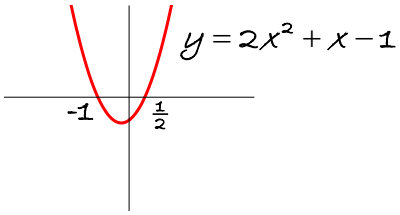
AQA FM Further algebra 3 Exercise solutions

(vii) $1 - x - 2x^2 \geq 0$

$2x^2 + x - 1 \leq 0$

$(2x - 1)(x + 1) \leq 0$

From graph, $-1 \leq x \leq \frac{1}{2}$

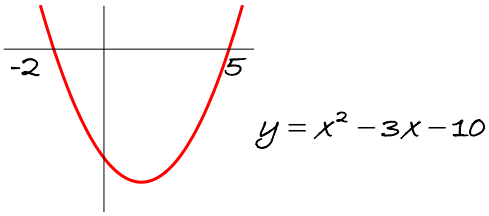


(ix) $x^2 \geq 3x + 10$

$x^2 - 3x - 10 \geq 0$

$(x - 5)(x + 2) \geq 0$

From graph, $x \leq -2$ or $x \geq 5$



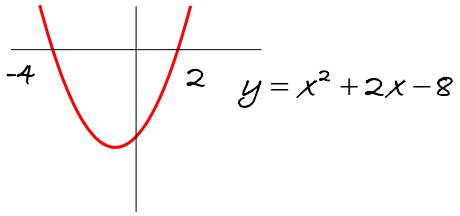
(x) $x(x + 3) > x + 8$

$x^2 + 3x > x + 8$

$x^2 + 2x - 8 > 0$

$(x + 4)(x - 2) > 0$

From graph, $x < -4$ or $x > 2$

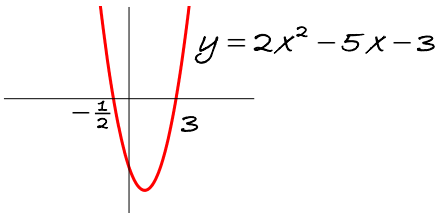


4. (i) $2x^2 - 5x - 3 \leq 0$

$(2x + 1)(x - 3) \leq 0$

From graph, $-\frac{1}{2} \leq x \leq 3$

The integer values are 0, 1, 2, 3



(ii) $x^2 + 2x - 1 < 0$ cannot be factorised, so use quadratic formula to solve the equation $x^2 + 2x - 1 = 0$:

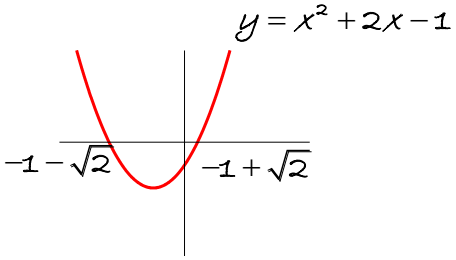
$a = 1, b = 2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -1}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

From graph, $-1 - \sqrt{2} < x < -1 + \sqrt{2}$

The integer values are -2, -1, 0.



5. (i) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(ii) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(iii) $4^{1/2} = \sqrt{4} = 2$

(iv) $6^0 = 1$

(v) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

(vi) $64^{1/3} = \sqrt[3]{64} = 4$

(vii) $16^{-1/2} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

(viii) $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

(ix) $36^{-3/2} = \frac{1}{\left(\sqrt{36}\right)^3} = \frac{1}{6^3} = \frac{1}{216}$

(x) $\left(\frac{1}{2}\right)^{-1} = \left(2^{-1}\right)^{-1} = 2^1 = 2$

(xi) $\left(\frac{25}{9}\right)^{-1/2} = \left(\frac{9}{25}\right)^{1/2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

(xii) $\left(\frac{27}{64}\right)^{-2/3} = \left(\frac{64}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{64}{27}}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

(xiii) $\frac{2^5 \times 4^{1/2}}{2} = \frac{2^5 \times (2^2)^{1/2}}{2} = \frac{2^5 \times 2^1}{2} = 2^{5+1-1} = 2^5 = 32$

(xiv) $\left(3^5\right)^{3/2} \times 9^{-7/4} = \left(3^5\right)^{3/2} \times \left(3^2\right)^{-7/4} = 3^{15/2} \times 3^{-7/2} = 3^{\frac{15}{2}-\frac{7}{2}} = 3^4 = 81$

6. (i) $2a^3b \times 3ab \times 5b^3 = 2 \times 3 \times 5 \times a^3 \times a \times b \times b \times b^3 = 30a^4b^5$

(ii) $\frac{2a^2b}{4ab^2} = \frac{\cancel{2} \times \cancel{a} \times a \times \cancel{b}}{\cancel{2}\cancel{4} \times \cancel{a} \times b \times \cancel{b}} = \frac{a}{2b}$

(iii) $\frac{12p^2qr^3}{9pq^2r} = \frac{4\cancel{12} \times \cancel{p} \times p \times \cancel{q} \times \cancel{r} \times r \times r}{3\cancel{9} \times \cancel{p} \times \cancel{q} \times q \times \cancel{r}} = \frac{4pr^2}{3q}$

(iv) $4xy^2 \div \left(2x^2y\right)^3 = \frac{\cancel{4} \times \cancel{y^2}}{2\cancel{8}x^{\cancel{6}5}y^{\cancel{3}1}} = \frac{1}{2x^5y}$

(v) $(a^{11} \times a^{-4} \div a^3 = a^{11-4-3} = a^4$

(vi) $\left(p^5\right)^3 \times \left(p^{\cancel{7}}\right)^{-2} = p^{15} \times p^{-14} = p^{15-14} = p$

(vii) $x^{\frac{3}{2}} \times \sqrt{x} = x^{\frac{3}{2}} \times x^{\frac{1}{2}} = x^{\frac{3}{2}+\frac{1}{2}} = x^2$

(viii) $\left(y^{\frac{1}{3}}\right)^2 \div y = y^{\frac{2}{3}} \times y^{-1} = y^{\frac{2}{3}-1} = y^{-\frac{1}{3}}$

(ix) $\frac{p^6}{p^5 \times p^3} = p^{6-5-3} = p^{-2} = \frac{1}{p^2}$

(x) $\sqrt{\frac{x^{4/3}}{x^{1/3} \times x^{8/3}}} = \sqrt{x^{\frac{4}{3}-\frac{1}{3}-\frac{8}{3}}} = \sqrt{x^{-\frac{5}{3}}} = \left(x^{-\frac{5}{3}}\right)^{\frac{1}{2}} = x^{-\frac{5}{6}}$

7. (i) $x^{\frac{3}{2}} = 8$

$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$

$x = 2^2 = 4$

(ii) $y^{-2} = \frac{9}{4}$

$y^2 = \frac{4}{9}$

$y = \sqrt{\frac{4}{9}} = \frac{2}{3}$

8. $(x+3)^2 > (x-1)^2$

$(x+3)^2 - (x-1)^2 > 0$

$(2x+2)(4) > 0$

$x + 1 > 0$

$x > -1$

9. $\sqrt{\frac{b}{d}} = \sqrt{\frac{x^a}{x^c}}$

$= \sqrt{x^{a-c}}$

$= x^{\frac{a-c}{2}}$