AQA Level 2 Further mathematics Further algebra

Section 2: Further equations

Solutions to Exercise

1. (i) $2x+5y=11$ (1)

$2x-y=5$ (2)

Subtracting: $6y=6$

$y=1$

Substituting into (1): $2x+5\times1=11$

$2x=6$

$x=3$

The solution is $x=3$, $y=1$. Check: $2x+5y=2\times3+5\times1=11$

$2x-y=2\times3-1=5$

(ii) $x+2y=6$ (1)×4 $4x+8y=24$

$4x+3y=4$ (2) $4x+3y=4$

Subtracting: $5y=20$

$y=4$

Substituting into (1): $x+2\times4=6$

$x=-2$

The solution is $x=$ -2, $y=4$. Check: $x+2y=-2+8=6$

$4x+3y=-8+12=4$

(iii) $3a-2b=4$ (1)×2 $6a-4b=8$

$5a+4b=3$ (2) $5a+4b=3$

Adding: $11a=11$

$a=1$

Substituting into (1): $3\times1-2b=4$

$-2b=1$

$b=-\frac{1}{2}$

The solution is $a=1$, $b=-\frac{1}{2}$. Check: $3a-2b=3+1=4$

$5a+4b=5-2=3$

(iv) $2p-5q=5$ (1)×3 $6p-15q=15$

$3p-2q=-9$ (2)×2 $6p-4q=-18$

Subtracting: $-11q=33$

$q=-3$

Substituting into (1): $2p-5\times-3=5$

$2p=-10$

$p=-5$

The solution is $p = -5, q = -3$. Check: $2p - 5q = -10 + 15 = 5$

$3p - 2q = -15 + 6 = -9$

(v) $5x + 3y = 9$ (1)

$y = 3x - 4$ (2)

Substituting (2) into (1): $5x + 3(3x - 4) = 9$

$5x + 9x - 12 = 9$

$14x = 21$

$x = \frac{3}{2}$

Substituting into (1): $y = 3 \times \frac{3}{2} - 4 = \frac{9}{2} - 4 = \frac{1}{2}$

The solution is $x = \frac{3}{2}, y = \frac{1}{2}$. Check: $5x + 3y = \frac{15}{2} + \frac{3}{2} = 9$

(vi) $3a + 2b = 1$ (1) $\times 2$ $\quad 6a + 4b = 2$

$9a - 4b = 4$ (2) $\quad 9a - 4b = 4$

Adding: $15a \quad = 6$

$a = \frac{2}{5}$

Substituting into (1): $3 \times \frac{2}{5} + 2b = 1$

$2b = 1 - \frac{6}{5} = -\frac{1}{5}$

$b = -\frac{1}{10}$

The solution is $a = \frac{2}{5}, b = -\frac{1}{10}$. Check: $3a + 2b = \frac{6}{5} - \frac{1}{5} = 1$

$9a - 4b = \frac{18}{5} + \frac{2}{5} = 4$

2. (i) $7x^2 + y^2 = 64$ (1)

$x + y = 4$ (2)

(2) $\Rightarrow y = 4 - x$

Substituting into (1): $7x^2 + (4 - x)^2 = 64$

$7x^2 + 16 - 8x + x^2 = 64$

$8x^2 - 8x - 48 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

When $x = 3$, $y = 4 - 3 = 1$

When $x = -2$, $y = 4 - (-2) = 6$

The solutions are $x = 3, y = 1$ and $x = -2, y = 6$

Check: $x = 3, y = 1 \Rightarrow 7x^2 + y^2 = 63 + 1 = 64$

$x = -2, y = 6 \Rightarrow 7x^2 + y^2 = 28 + 36 = 64$

(ii) $3x^2 - 2y^2 = -5$ (1)

$y - x = 1$ (2)

(2) $\Rightarrow y = 1 + x$

Substituting into (1): $3x^2 - 2(1 + x)^2 = -5$

$3x^2 - 2(1 + 2x + x^2) = -5$

$3x^2 - 2 - 4x - 2x^2 = -5$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

When $x = 1$, $y = 1 + 1 = 2$

When $x = 3$, $y = 1 + 3 = 4$

The solutions are $x = 1, y = 2$ and $x = 3, y = 4$

Check: $x = 1, y = 2 \Rightarrow 3x^2 - 2y^2 = 3 - 5 = -5$

$x = 3, y = 4 \Rightarrow 3x^2 - 2y^2 = 27 - 32 = -5$

(iii) $p^2 + pq = 2$ (1)

$q - p = 3$ (2)

(2) $\Rightarrow q = 3 + p$

Substituting into (1): $p^2 + p(3 + p) = 2$

$p^2 + 3p + p^2 = 2$

$2p^2 + 3p - 2 = 0$

$(2p - 1)(p + 2) = 0$

$p = \frac{1}{2}$ or $p = -2$

When $p = \frac{1}{2}$, $q = 3 + \frac{1}{2} = \frac{7}{2}$

When $p = -2$, $q = 3 - 2 = 1$

The solutions are $p = \frac{1}{2}, q = \frac{7}{2}$ and $p = -2, q = 1$.

Check: $p = \frac{1}{2}, q = \frac{7}{2} \Rightarrow p^2 + pq = \frac{1}{4} + \frac{7}{4} = 2$

$p = -2, q = 1 \Rightarrow p^2 + pq = 4 - 2 = 2$

(iv) $8a^2 - b^2 = 2$ (1)

$2a + b = 1$ (2)

(2) $\Rightarrow b = 1 - 2a$

Substituting into (1): $8a^2-(1-2a)^2=2$

$$8a^2-(1-4a+4a^2)=2$$
$$8a^2-1+4a-4a^2=2$$
$$4a^2+4a-3=0$$
$$(2a+3)(2a-1)=0$$
$$a=-\tfrac{3}{2} \text{ or } a=\tfrac{1}{2}$$

When $a=-\tfrac{3}{2}$, $b=1-2\times-\tfrac{3}{2}=1+3=4$

When $a=\tfrac{1}{2}$, $b=1-2\times\tfrac{1}{2}=1-1=0$

The solutions are $a=-\tfrac{3}{2}, b=4$ and $a=\tfrac{1}{2}, b=0$.

Check: $a=-\tfrac{3}{2}, b=4 \Rightarrow 8a^2-b^2=8\times\tfrac{9}{4}-16=18-16=2$

$a=\tfrac{1}{2}, b=0 \Rightarrow 8a^2-b^2=8\times\tfrac{1}{4}-0=2$

3. (i) $f(x)=x^3-4x^2+x+6$

$f(-1)=(-1)^3-4(-1)^2+(-1)+6=-1-4-1+6=0$

so by the factor theorem, $x+1$ is a factor.

(ii) $x^3-4x^2+x+6=(x+1)(x^2-5x+6)$

$=(x+1)(x-2)(x-3)$

4. (i) $f(x)=x^3+ax^2-4x+12$

$f(2)=2^3+a\times2^2-4\times2+12$

$=8+4a-8+12$

$=4a+12$

$x-2$ is a factor so by the factor theorem $f(2)=0$

$4a+12=0$

$a=-3$

(ii) $x^3-3x^2-4x+12=(x-2)(x^2-x-6)$

$=(x-2)(x+2)(x-3)$

5. (i) $f(x)=x^3-2x^2-11x+12$

$f(1)=1-2-11+12=0$ so $(x-1)$ is a factor

$x^3-2x^2-11x+12=0$

$(x-1)(x^2-x-12)=0$

$(x-1)(x+3)(x-4)=0$

$x = 1, x = -3, x = 4$

(ii) $f(x) = x^3 + 4x^2 - 3x - 18$

$f(1) = 1 + 4 - 3 - 18 \neq 0$

$f(-1) = -1 + 4 + 3 - 18 \neq 0$

$f(2) = 8 + 16 - 6 - 18 = 0$

so $(x - 2)$ is a factor

$x^3 + 4x^2 - 3x - 18 = 0$

$(x - 2)(x^2 + 6x + 9) = 0$

$(x - 2)(x + 3)^2 = 0$

$x = 2$ or $x = -3$

(iii) $f(x) = x^3 - 19x - 30$

$f(1) = 1 - 19 - 30 \neq 0$

$f(2) = 8 - 38 - 30 \neq 0$

$f(-2) = -8 + 38 - 30 = 0$

so $x + 2$ is a factor

$x^3 - 19x - 30 = 0$

$(x + 2)(x^2 - 2x - 15) = 0$

$(x + 2)(x + 3)(x - 5) = 0$

$x = -2$ or $x = -3$ or $x = 5$

6. If Bob is right, then $x = 1$, $x = 2$ and $x = -5$ would all make $x^3 - 4x^2 - 7x + 10$ be zero.

<u>$x=1$</u>

$x^3 - 4x^2 - 7x + 10 = 1 - 4 - 7 + 10 = 0$ so $(x - 1)$ is a factor.

<u>$x=2$</u>

$x^3 - 4x^2 - 7x + 10 = 8 - 16 - 14 + 10 = -12 \neq 0$ so $(x - 2)$ is not a factor.

7. $xy - 9 = 15$

$2x + 2y = 20$

$y = 10 - x$

$x(10 - x) = 24$

$10x - x^2 = 24$

$x^2 - 10x + 24 = 0$

$(x - 6)(x - 4) = 10$

$x = 6$ and $y = 4$ (or $x = 4$ and $y = 6$)