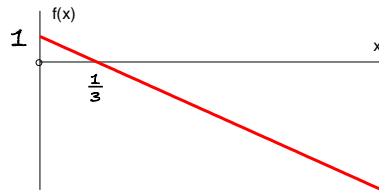


Section 3: Functions and their graphs

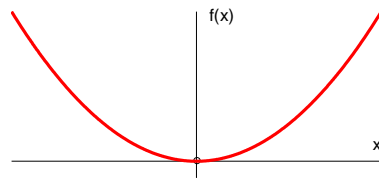
Solutions to Exercise

1. (i) $y = 1 - 3x$ where $x > 0$
When $x = 0$, $y = 1$
When $y = 0$, $1 - 3x = 0 \Rightarrow x = \frac{1}{3}$



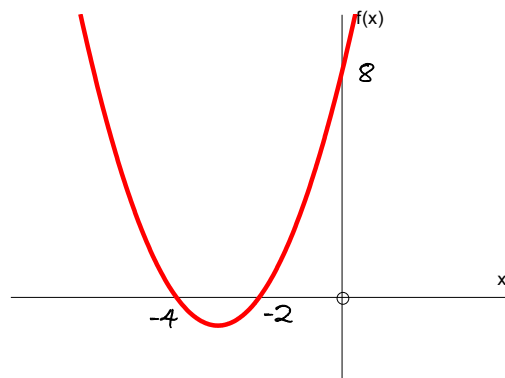
The range is $f(x) < 1$.

- (ii) $y = x^2$ where x can take any value
When $x = 0$, $y = 0$



The range is $f(x) \geq 0$.

2. (i) $y = x^2 + 6x + 8$
When $x = 0$, $y = 8$
When $y = 0$, $x^2 + 6x + 8 = 0$
 $(x+2)(x+4) = 0$
 $x = -2$ or -4



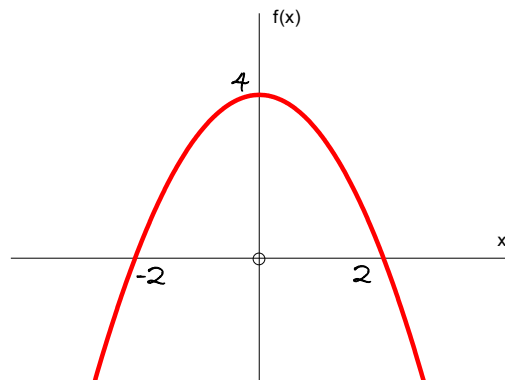
- (ii) $y = 4 - x^2$
When $x = 0$, $y = 4$

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$$\text{When } y = 0, 4 - x^2 = 0$$

$$(2 - x)(2 + x) = 0$$

$$x = 2 \text{ or } -2$$



$$3. \text{ (i) } f(-1) = \frac{1}{1 + (-1)^2} = \frac{1}{2} = 0.5$$

$$f\left(\frac{1}{2}\right) = \frac{1}{1 + \left(\frac{1}{2}\right)^2} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = 0.8$$

$$\text{(ii) } f(x) = \frac{1}{1 + x^2} \text{ where } -1 \leq x \leq 1$$

The largest possible value of $f(x)$ is when $x = 0$, where $f(x) = 1$.

The smallest possible value of $f(x)$ is when $x = \pm 1$, where $f(x) = \frac{1}{2}$.

The range is $\frac{1}{2} \leq f(x) \leq 1$.

4. (i) $x = 1$ must be excluded from the domain, since the function is not defined for this value.

$$\text{(ii) (a) } f(2) = \frac{1}{2 - 1} = 1$$

$$\text{(b) } f(-3) = \frac{1}{-3 - 1} = -\frac{1}{4}$$

$$\text{(c) } f(0) = \frac{1}{0 - 1} = -1$$

$$\text{(iii) } f(x) = 2$$

$$\frac{1}{x - 1} = 2$$

$$1 = 2(x - 1)$$

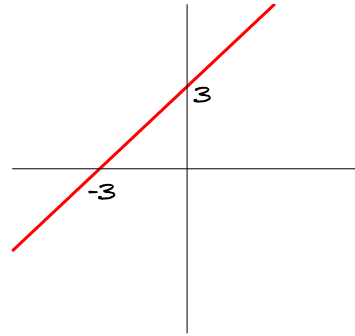
$$1 = 2x - 2$$

$$2x = 3$$

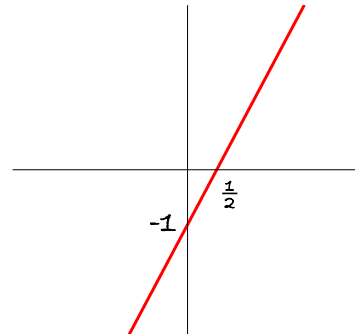
$$x = \frac{3}{2}$$

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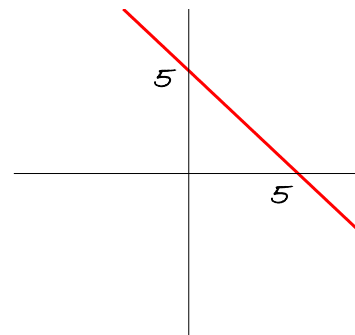
5. (i) $y = x + 3$
Gradient = 1
When $x = 0$, $y = 3$
When $y = 0$, $x + 3 = 0 \Rightarrow x = -3$



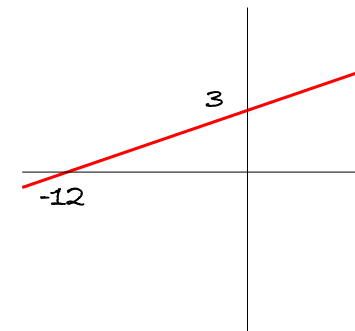
(ii) $y = 2x - 1$
Gradient = 2
When $x = 0$, $y = -1$
When $y = 0$, $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$



(iii) $x + y = 5$
 $y = -x + 5$
Gradient = -1
When $x = 0$, $y = 5$
When $y = 0$, $x = 5$



(iv) $4y = x + 12$
 $y = \frac{1}{4}x + 3$
Gradient = $\frac{1}{4}$
When $x = 0$, $y = 3$
When $y = 0$, $x + 12 = 0 \Rightarrow x = -12$



(v) $3y + x + 6 = 0$
 $3y = -x - 6$
 $y = -\frac{1}{3}x - 2$
Gradient = $-\frac{1}{3}$
When $x = 0$, $y = -2$
When $y = 0$, $x + 6 = 0 \Rightarrow x = -6$

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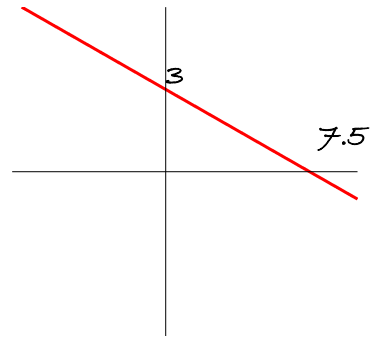
$$(vi) 5y = 15 - 2x$$

$$y = 3 - \frac{2}{5}x$$

$$\text{Gradient} = -\frac{2}{5}$$

$$\text{When } x = 0, 5y = 15 \Rightarrow y = 3$$

$$\text{When } y = 0, 15 - 2x = 0 \Rightarrow x = 7.5$$



6. (a) Gradient = 1, y-intercept = 2
Equation of line is $y = x + 2$

(b) Gradient = $\frac{1}{2}$, y-intercept = -1
Equation of line is $y = \frac{1}{2}x - 1$
or $2y = x - 2$

(c) Gradient = $-\frac{1}{2}$, y-intercept = -2
Equation of line is $y = -\frac{1}{2}x - 2$
or $2y + x + 4 = 0$

(d) Gradient = $-\frac{1}{4}$, y-intercept = 3
Equation of line is $y = -\frac{1}{4}x + 3$
or $4y + x = 12$

(e) Gradient = $-\frac{8}{3}$, passes through (-1, 4)
Equation of line is $y - 4 = -\frac{8}{3}(x - (-1))$
 $3(y - 4) = -8(x + 1)$
 $3y - 12 = -8x - 8$
 $3y + 8x = 4$

7. (i) Equation of line is $y - 3 = 4(x - 2)$
 $y - 3 = 4x - 8$
 $y = 4x - 5$

(ii) Equation of line is $y - (-1) = -\frac{1}{3}(x - 4)$
 $3(y + 1) = -(x - 4)$
 $3y + 3 = -x + 4$
 $3y + x = 1$

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(iii) Equation of line is $y - (-6) = -\frac{1}{5}(x - (-1))$
 $5(y + 6) = -(x + 1)$
 $5y + 30 = -x - 1$
 $5y + x + 31 = 0$

8. (i) Gradient of AB = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - 2}{1 - 3} = \frac{4}{-2} = -2$
Equation of AB is $y - 6 = -2(x - 1)$
 $y - 6 = -2x + 2$
 $y + 2x = 8$

(ii) Gradient of AB = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{-1 - 3}{8 - (-2)} = \frac{-4}{10} = -\frac{2}{5}$
Equation of AB is $y - (-1) = -\frac{2}{5}(x - 8)$
 $5(y + 1) = -2(x - 8)$
 $5y + 5 = -2x + 16$
 $5y + 2x = 11$

(iii) Gradient of AB = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - (-4)}{-5 - 7} = \frac{6}{-12} = -\frac{1}{2}$
Equation of AB is $y - 2 = -\frac{1}{2}(x - (-5))$
 $2(y - 2) = -(x + 5)$
 $2y - 4 = -x - 5$
 $2y + x + 1 = 0$

(iv) Gradient of AB = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{-5 - 1}{-3 - 5} = \frac{-6}{-8} = \frac{3}{4}$
Equation of AB is $y - (-5) = \frac{3}{4}(x - (-3))$
 $4(y + 5) = 3(x + 3)$
 $4y + 20 = 3x + 9$
 $4y = 3x - 11$

9. Let the triangle be ABC.

Let A be the intersection point of $y + 3x = 11$ and $3y = x + 3$.

$$y + 3x = 11 \Rightarrow y = 11 - 3x$$

Substituting into $3y = x + 3$ gives $3(11 - 3x) = x + 3$

$$33 - 9x = x + 3$$

$$30 = 10x$$

$$x = 3$$

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$$\text{When } x = 3, y = 11 - 3 \times 3 = 2$$

The coordinates of A are (3, 2).

Let B be the intersection point of $3y = x + 3$ and $7y + x = 37$

$$3y = x + 3 \Rightarrow x = 3y - 3$$

Substituting into $7y + x = 37$ gives $7y + 3y - 3 = 37$

$$10y = 40$$

$$y = 4$$

$$\text{When } y = 4, x = 3 \times 4 - 3 = 9$$

The coordinates of B are (9, 4).

Let C be the intersection point of $7y + x = 37$ and $y + 3x = 11$

$$y + 3x = 11 \Rightarrow y = 11 - 3x$$

Substituting into $7y + x = 37$ gives $7(11 - 3x) + x = 37$

$$77 - 21x + x = 37$$

$$40 = 20x$$

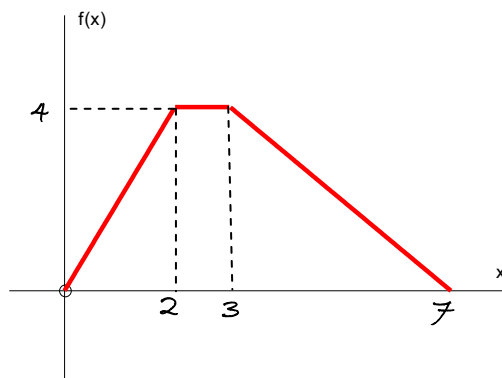
$$x = 2$$

$$\text{When } x = 2, y = 11 - 3 \times 2 = 5$$

The coordinates of C are (2, 5).

The vertices of the triangle are (3, 2), (9, 4) and (2, 5).

10.



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$$\begin{aligned} 11. \frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h} \\ &= \frac{3(x+h)^2 - h - 3x^2}{h} \\ &= \frac{3\{(x+h)^2 - x^2\} - h}{h} \\ &= \frac{3(x+h-x)(x+h+x) - h}{h} \\ &= \frac{3h(2x+h) - h}{h} \\ &= 3(2x+h) - 1 \\ &= 6x + 3h - 1 \end{aligned}$$

12. One possible function is $f(x) = x^2 + 2$.