

AQA Level 2 Further mathematics Number & algebra

Section 1: Basic number and algebra

Solutions to Exercise

$$\begin{aligned} 1. \quad (i) \quad 3\frac{3}{4} - 2\frac{2}{3} &= 1 + \frac{3}{4} - \frac{2}{3} \\ &= 1 + \frac{9-8}{12} \\ &= 1\frac{1}{12} \end{aligned}$$

$$(ii) \quad 1\frac{2}{5} \times 2\frac{1}{3} = \frac{7}{5} \times \frac{7}{3} = \frac{49}{15} = 3\frac{4}{15}$$

$$(iii) \quad 3\frac{3}{5} \div 2\frac{2}{3} = \frac{18}{5} \div \frac{8}{3} = \frac{18}{5} \times \frac{3}{8} = \frac{9}{5} \times \frac{3}{4} = \frac{27}{20} = 1\frac{7}{20}$$

$$\begin{aligned} 2. \quad (i) \quad x : z &= 2 : 5 = 6 : 15 \\ y : z &= 3 : 4 = 15 : 20 \\ x : z &= 6 : 20 = 3 : 10 \end{aligned}$$

$$(ii) \quad 2y : 5z = 6 : 20 = 3 : 10$$

$$(iii) \quad x + 2y : y = 12 : 5$$

$$3. \quad x : y = y : 4$$

$$\frac{x}{y} = \frac{y}{4}$$

$$4x = y^2$$

$$(a) \quad x = y = 4$$

$$(b) \quad x = 25, y = 10 \text{ is one possible pair}$$

$$(c) \quad x = 1, y = 2$$

$$\begin{aligned} 4. \quad (i) \quad \text{Increasing by } 20\% \text{ is equivalent to multiplying by } 1.2 \\ 230 \times 1.2 &= 276 \\ \text{The price is } \pounds 276. \end{aligned}$$

$$(ii) \quad \frac{680}{800} = 0.85, \text{ so the price has been multiplied by } 0.85. \text{ So the new price is } 85\% \text{ of the old price, and so the price has been reduced by } 15\%.$$

$$\begin{aligned} 5. \quad (i) \quad 2x + 3y - x + 5y + 4x &= (2x - x + 4x) + (3y + 5y) \\ &= 5x + 8y \end{aligned}$$

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$$\begin{aligned} \text{(ii)} \quad 5a - 2b + 3c - 2a + 5b &= (5a - 2a) + (-2b + 5b) + 3c \\ &= 3a + 3b + 3c \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad 4p + q - 6p - 5q + 5p + 4q &= (4p - 6p + 5p) + (q - 5q + 4q) \\ &= 3p \end{aligned}$$

$$6. \text{ (i)} \quad 3(2x + 3y) = 6x + 9y$$

$$\begin{aligned} \text{(ii)} \quad 4(3a - 2b) - 3(a + 2b) &= 12a - 8b - 3a - 6b \\ &= 9a - 14b \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad p(2p - q) + 2q(p - 3q) &= 2p^2 - pq + 2qp - 6q^2 \\ &= 2p^2 + pq - 6q^2 \end{aligned}$$

$$\begin{aligned} 7. \text{ (i)} \quad (x + 1)(x - 3) &= x^2 - 3x + x - 3 \\ &= x^2 - 2x - 3 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (x + 2)(2x + 1) &= 2x^2 + x + 4x + 2 \\ &= 2x^2 + 5x + 2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (x - 3)(x - 4) &= x^2 - 4x - 3x + 12 \\ &= x^2 - 7x + 12 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (3x + 2)(x - 2) &= 3x^2 - 6x + 2x - 4 \\ &= 3x^2 - 4x - 4 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (2x + 1)(4x - 1) &= 8x^2 - 2x + 4x - 1 \\ &= 8x^2 + 2x - 1 \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad (1 - 2x)(1 + x) &= 1 + x - 2x - 2x^2 \\ &= 1 - x - 2x^2 \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad (3 + 2x)(x - 1) &= 3x - 3 + 2x^2 - 2x \\ &= 2x^2 + x - 3 \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad (5x - 3)(2x + 5) &= 10x^2 + 25x - 6x - 15 \\ &= 10x^2 + 19x - 15 \end{aligned}$$

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$$\begin{aligned} \text{(ix)} \quad (x+3)^3 &= (x^2+6x+9)(x+3) \\ &= x^3+6x^2+9x+3x^2+18x+27 \\ &= x^3+9x^2+27x+27 \end{aligned}$$

$$\begin{aligned} 8. \text{ (i)} \quad (x-2)(2x^2-3x+1) &= x(2x^2-3x+1)-2(2x^2-3x+1) \\ &= 2x^3-3x^2+x-4x^2+6x-2 \\ &= 2x^3-7x^2+7x-2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (3x-2)(x^3-2x+4) &= 3x(x^3-2x+4)-2(x^3-2x+4) \\ &= 3x^4-6x^2+12x-2x^3+4x-8 \\ &= 3x^4-2x^3-6x^2+16x-8 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (2x+1)(x^3+2x^2-3x-5) \\ &= 2x(x^3+2x^2-3x-5)+(x^3+2x^2-3x-5) \\ &= 2x^4+4x^3-6x^2-10x+x^3+2x^2-3x-5 \\ &= 2x^4+5x^3-4x^2-13x-5 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (x+3)(2x-1)(x-4) &= (x+3)(2x^2-8x-x+4) \\ &= (x+3)(2x^2-9x+4) \\ &= x(2x^2-9x+4)+3(2x^2-9x+4) \\ &= 2x^3-9x^2+4x+6x^2-27x+12 \\ &= 2x^3-3x^2-23x+12 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (2x-1)^3 &= (4x^2-4x+1)(2x-1) \\ &= 8x^3-8x^2+2x-4x^2+4x-1 \\ &= 8x^3-12x^2+6x-1 \end{aligned}$$

$$9. \text{ (i)} \quad \sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

$$\text{(ii)} \quad \sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

$$\text{(iii)} \quad \sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

$$\text{(iv)} \quad \sqrt{216} = \sqrt{36 \times 6} = \sqrt{36} \times \sqrt{6} = 6\sqrt{6}$$

$$\text{(v)} \quad \sqrt{63} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$$

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$$(vi) \quad \sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$$

$$(vii) \quad \sqrt{6} \times \sqrt{27} = \sqrt{3} \times \sqrt{2} \times \sqrt{9} \times \sqrt{3} = \sqrt{2} \times 3 \times 3 = 9\sqrt{2}$$

$$(viii) \quad \sqrt{12} \times \sqrt{15} = \sqrt{4} \times \sqrt{3} \times \sqrt{3} \times \sqrt{5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}$$

$$(ix) \quad \begin{aligned} \sqrt{10} \times \sqrt{24} \times \sqrt{15} &= \sqrt{5} \times \sqrt{2} \times \sqrt{4} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3} \times \sqrt{5} \\ &= 5 \times 2 \times 2 \times 3 \\ &= 60 \end{aligned}$$

$$10. (i) \quad \begin{aligned} (1 + \sqrt{2}) + (3 - 2\sqrt{2}) &= 1 + 3 + \sqrt{2} - 2\sqrt{2} \\ &= 4 - \sqrt{2} \end{aligned}$$

$$(ii) \quad \begin{aligned} (5\sqrt{2} - 2\sqrt{3}) - (\sqrt{2} + 3\sqrt{3}) &= 5\sqrt{2} - 2\sqrt{3} - \sqrt{2} - 3\sqrt{3} \\ &= 4\sqrt{2} - 5\sqrt{3} \end{aligned}$$

$$(iii) \quad \begin{aligned} 2(\sqrt{5} - 3\sqrt{3}) + 3(2\sqrt{5} + \sqrt{3}) &= 2\sqrt{5} - 6\sqrt{3} + 6\sqrt{5} + 3\sqrt{3} \\ &= 8\sqrt{5} - 3\sqrt{3} \end{aligned}$$

$$(iv) \quad \begin{aligned} \sqrt{18} + \sqrt{72} - \sqrt{98} &= \sqrt{9 \times 2} + \sqrt{36 \times 2} - \sqrt{49 \times 2} \\ &= 3\sqrt{2} + 6\sqrt{2} - 7\sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$11. (i) \quad \begin{aligned} (1 + \sqrt{2})(3 - \sqrt{2}) &= 3 - \sqrt{2} + 3\sqrt{2} - 2 \\ &= 1 + 2\sqrt{2} \end{aligned}$$

$$(ii) \quad \begin{aligned} (2 - \sqrt{3})(3 + 2\sqrt{3}) &= 6 + 4\sqrt{3} - 3\sqrt{3} - 2 \times 3 \\ &= \sqrt{3} \end{aligned}$$

$$(iii) \quad \begin{aligned} (3 - 2\sqrt{5})(1 - 3\sqrt{5}) &= 3 - 9\sqrt{5} - 2\sqrt{5} + 6 \times 5 \\ &= 33 - 11\sqrt{5} \end{aligned}$$

$$(iv) \quad \begin{aligned} (\sqrt{2} + 2\sqrt{3})(5\sqrt{2} - \sqrt{3}) &= 5 \times 2 - \sqrt{2}\sqrt{3} + 10\sqrt{3}\sqrt{2} - 2 \times 3 \\ &= 10 - \sqrt{6} + 10\sqrt{6} - 6 \\ &= 4 + 9\sqrt{6} \end{aligned}$$

$$(v) \quad \begin{aligned} (\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) &= 7 - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - 2 \\ &= 5 \end{aligned}$$

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$$\begin{aligned}(\text{vi}) \quad (3-\sqrt{2})^2 &= (3-\sqrt{2})(3-\sqrt{2}) \\ &= 9-3\sqrt{2}-3\sqrt{2}+2 \\ &= 11-6\sqrt{2}\end{aligned}$$

$$12. \text{ (i)} \quad \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\text{(ii)} \quad \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\text{(iii)} \quad \frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(1+\sqrt{2})\sqrt{2}}{2} = \frac{\sqrt{2}+2}{2}$$

$$\text{(iv)} \quad \frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}-1}{3-1} = \frac{\sqrt{3}-1}{2}$$

$$\text{(v)} \quad \frac{\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{2}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{2\sqrt{2}+2}{4-2} = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$$

$$\begin{aligned}\text{(vi)} \quad \frac{1-\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} &= \frac{(1-\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \\ &= \frac{2+\sqrt{3}-2\sqrt{3}-3}{4-3} \\ &= \frac{-1-\sqrt{3}}{1} \\ &= -1-\sqrt{3}\end{aligned}$$

$$\begin{aligned}\text{(vii)} \quad \frac{1+2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} &= \frac{(1+2\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\ &= \frac{3+\sqrt{5}+6\sqrt{5}+2 \times 5}{9-5} \\ &= \frac{13+7\sqrt{5}}{4}\end{aligned}$$

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$$\begin{aligned} \text{(viii)} \quad \frac{1+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} &= \frac{(1+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\ &= \frac{\sqrt{3}-\sqrt{2}+\sqrt{6}-2}{3-2} \\ &= \sqrt{3}-\sqrt{2}+\sqrt{6}-2 \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} &= \frac{(\sqrt{6}+\sqrt{3})(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} \\ &= \frac{6+\sqrt{18}+\sqrt{18}+3}{6-3} \\ &= \frac{9+2\sqrt{18}}{3} \end{aligned}$$

13. One possible width is $(\sqrt{7}-1)$ cm.

In this case, the area is $(\sqrt{7}+1)(\sqrt{7}-1) = 7-1 = 6 \text{ cm}^2$.

Other answers are possible.