# AQA Level 2 Further mathematics Number & algebra

## Section 1: Basic number and algebra

### Solutions to Exercise

1. (i) $3\frac{3}{4} - 2\frac{2}{3} = 1 + \frac{3}{4} - \frac{2}{3}$

$= 1 + \frac{9-8}{12}$

$= 1\frac{1}{12}$

(ii) $1\frac{2}{5} \times 2\frac{1}{3} = \frac{7}{5} \times \frac{7}{3} = \frac{49}{15} = 3\frac{4}{15}$

(iii) $3\frac{3}{5} \div 2\frac{2}{3} = \frac{18}{5} \div \frac{8}{3} = \frac{18}{5} \times \frac{3}{8} = \frac{9}{5} \times \frac{3}{4} = \frac{27}{20} = 1\frac{7}{20}$

2. (i) $x : z = 2 : 5 = 6 : 15$

$y : z = 3 : 4 = 15 : 20$

$x : z = 6 : 20 = 3 : 10$

(ii) $2y : 5z = 6 : 20 = 3 : 10$

(iii) $x + 2y : y = 12 : 5$

3. $x : y = y : 4$

$\frac{x}{y} = \frac{y}{4}$

$4x = y^2$

(a) $x = y = 4$

(b) $x = 25$, $y = 10$ is one possible pair

(c) $x = 1$, $y = 2$

4. (i) Increasing by 20% is equivalent to multiplying by 1.2

$230 \times 1.2 = 276$

The price is £276.

(ii) $\frac{680}{800} = 0.85$, so the price has been multiplied by 0.85. So the new price is 85% of the old price, and so the price has been reduced by 15%.

5. (i) $2x + 3y - x + 5y + 4x = (2x - x + 4x) + (3y + 5y)$

$= 5x + 8y$

(ii) $5a-2b+3c-2a+5b=(5a-2a)+(-2b+5b)+3c$
$=3a+3b+3c$

(iii) $4p+q-6p-5q+5p+4q=(4p-6p+5p)+(q-5q+4q)$
$=3p$

6. (i) $3(2x+3y)=6x+9y$

(ii) $4(3a-2b)-3(a+2b)=12a-8b-3a-6b$
$=9a-14b$

(iii) $p(2p-q)+2q(p-3q)=2p^2-pq+2qp-6q^2$
$=2p^2+pq-6q^2$

7. (i) $(x+1)(x-3)=x^2-3x+x-3$
$=x^2-2x-3$

(ii) $(x+2)(2x+1)=2x^2+x+4x+2$
$=2x^2+5x+2$

(iii) $(x-3)(x-4)=x^2-4x-3x+12$
$=x^2-7x+12$

(iv) $(3x+2)(x-2)=3x^2-6x+2x-4$
$=3x^2-4x-4$

(v) $(2x+1)(4x-1)=8x^2-2x+4x-1$
$=8x^2+2x-1$

(vi) $(1-2x)(1+x)=1+x-2x-2x^2$
$=1-x-2x^2$

(vii) $(3+2x)(x-1)=3x-3+2x^2-2x$
$=2x^2+x-3$

(viii) $(5x-3)(2x+5)=10x^2+25x-6x-15$
$=10x^2+19x-15$

(ix) $(x+3)^3 = (x^2+6x+9)(x+3)$

$= x^3+6x^2+9x+3x^2+18x+27$

$= x^3+9x^2+27x+27$

8. (i) $(x-2)(2x^2-3x+1) = x(2x^2-3x+1)-2(2x^2-3x+1)$

$= 2x^3-3x^2+x-4x^2+6x-2$

$= 2x^3-7x^2+7x-2$

(ii) $(3x-2)(x^3-2x+4) = 3x(x^3-2x+4)-2(x^3-2x+4)$

$= 3x^4-6x^2+12x-2x^3+4x-8$

$= 3x^4-2x^3-6x^2+16x-8$

(iii) $(2x+1)(x^3+2x^2-3x-5)$

$= 2x(x^3+2x^2-3x-5)+(x^3+2x^2-3x-5)$

$= 2x^4+4x^3-6x^2-10x+x^3+2x^2-3x-5$

$= 2x^4+5x^3-4x^2-13x-5$

(iv) $(x+3)(2x-1)(x-4) = (x+3)(2x^2-8x-x+4)$

$= (x+3)(2x^2-9x+4)$

$= x(2x^2-9x+4)+3(2x^2-9x+4)$

$= 2x^3-9x^2+4x+6x^2-27x+12$

$= 2x^3-3x^2-23x+12$

(v) $(2x-1)^3 = (4x^2-4x+1)(2x-1)$

$= 8x^3-8x^2+2x-4x^2+4x-1$

$= 8x^3-12x^2+6x-1$

9. (i) $\sqrt{8} = \sqrt{4\times 2} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

(ii) $\sqrt{50} = \sqrt{25\times 2} = \sqrt{25}\times\sqrt{2} = 5\sqrt{2}$

(iii) $\sqrt{48} = \sqrt{16\times 3} = \sqrt{16}\times\sqrt{3} = 4\sqrt{3}$

(iv) $\sqrt{216} = \sqrt{36\times 6} = \sqrt{36}\times\sqrt{6} = 6\sqrt{6}$

(v) $\sqrt{63} = \sqrt{9\times 7} = \sqrt{9}\times\sqrt{7} = 3\sqrt{7}$

(vi) $\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

(vii) $\sqrt{6} \times \sqrt{27} = \sqrt{3} \times \sqrt{2} \times \sqrt{9} \times \sqrt{3} = \sqrt{2} \times 3 \times 3 = 9\sqrt{2}$

(viii) $\sqrt{12} \times \sqrt{15} = \sqrt{4} \times \sqrt{3} \times \sqrt{3} \times \sqrt{5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}$

(ix) $\sqrt{10} \times \sqrt{24} \times \sqrt{15} = \sqrt{5} \times \sqrt{2} \times \sqrt{4} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3} \times \sqrt{5}$
$= 5 \times 2 \times 2 \times 3$
$= 60$

10. (i) $(1 + \sqrt{2}) + (3 - 2\sqrt{2}) = 1 + 3 + \sqrt{2} - 2\sqrt{2}$
$= 4 - \sqrt{2}$

(ii) $(5\sqrt{2} - 2\sqrt{3}) - (\sqrt{2} + 3\sqrt{3}) = 5\sqrt{2} - 2\sqrt{3} - \sqrt{2} - 3\sqrt{3}$
$= 4\sqrt{2} - 5\sqrt{3}$

(iii) $2(\sqrt{5} - 3\sqrt{3}) + 3(2\sqrt{5} + \sqrt{3}) = 2\sqrt{5} - 6\sqrt{3} + 6\sqrt{5} + 3\sqrt{3}$
$= 8\sqrt{5} - 3\sqrt{3}$

(iv) $\sqrt{18} + \sqrt{72} - \sqrt{98} = \sqrt{9 \times 2} + \sqrt{36 \times 2} - \sqrt{49 \times 2}$
$= 3\sqrt{2} + 6\sqrt{2} - 7\sqrt{2}$
$= 2\sqrt{2}$

11. (i) $(1 + \sqrt{2})(3 - \sqrt{2}) = 3 - \sqrt{2} + 3\sqrt{2} - 2$
$= 1 + 2\sqrt{2}$

(ii) $(2 - \sqrt{3})(3 + 2\sqrt{3}) = 6 + 4\sqrt{3} - 3\sqrt{3} - 2 \times 3$
$= \sqrt{3}$

(iii) $(3 - 2\sqrt{5})(1 - 3\sqrt{5}) = 3 - 9\sqrt{5} - 2\sqrt{5} + 6 \times 5$
$= 33 - 11\sqrt{5}$

(iv) $(\sqrt{2} + 2\sqrt{3})(5\sqrt{2} - \sqrt{3}) = 5 \times 2 - \sqrt{2}\sqrt{3} + 10\sqrt{3}\sqrt{2} - 2 \times 3$
$= 10 - \sqrt{6} + 10\sqrt{6} - 6$
$= 4 + 9\sqrt{6}$

(v) $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) = 7 - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - 2$
$= 5$

(vi) $\left(3-\sqrt{2}\right)^2 = \left(3-\sqrt{2}\right)\left(3-\sqrt{2}\right)$
$= 9-3\sqrt{2}-3\sqrt{2}+2$
$= 11-6\sqrt{2}$

12. (i) $\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

(ii) $\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

(iii) $\frac{1+\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\left(1+\sqrt{2}\right)\sqrt{2}}{2} = \frac{\sqrt{2}+2}{2}$

(iv) $\frac{1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\sqrt{3}-1}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{\sqrt{3}-1}{3-1} = \frac{\sqrt{3}-1}{2}$

(v) $\frac{\sqrt{2}}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{2}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)} = \frac{2\sqrt{2}+2}{4-2} = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$

(vi) $\frac{1-\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{\left(1-\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$
$= \frac{2+\sqrt{3}-2\sqrt{3}-3}{4-3}$
$= \frac{-1-\sqrt{3}}{1}$
$= -1-\sqrt{3}$

(vii) $\frac{1+2\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{\left(1+2\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$
$= \frac{3+\sqrt{5}+6\sqrt{5}+2\times5}{9-5}$
$= \frac{13+7\sqrt{5}}{4}$

(viii) $\dfrac{1+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{(1+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \dfrac{\sqrt{3}-\sqrt{2}+\sqrt{6}-2}{3-2}$

$= \sqrt{3}-\sqrt{2}+\sqrt{6}-2$

(ix) $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}} \times \dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \dfrac{(\sqrt{6}+\sqrt{3})(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}$

$= \dfrac{6+\sqrt{18}+\sqrt{18}+3}{6-3}$

$= \dfrac{9+2\sqrt{18}}{3}$

13. One possible width is $(\sqrt{7}-1)$ cm.

In this case, the area is $(\sqrt{7}+1)(\sqrt{7}-1) = 7-1 = 6$ cm$^2$.

Other answers are possible.