AQA Level 2 Further mathematics Number & algebra

Section 2: Algebraic manipulation

Solutions to Exercise

1. (i) $10ab+5ac=5a(2b+c)$

(ii) $2x^2+4xy-8xz=2x(x+2y-4z)$

(iii) $3s^2t-9s^3t+12s^2t^2=3s^2t(1-3s+4t)$

(iv) $3(b-c)-2a(b-c)=(b-c)(3-2a)$

2. (i) $x^2+5x+6=x^2+3x+2x+6$
$=x(x+3)+2(x+3)$
$=(x+2)(x+3)$

(ii) $x^2+x-12=x^2+4x-3x-12$
$=x(x+4)-3(x+4)$
$=(x-3)(x+4)$

(iii) $x^2-9=(x+3)(x-3)$

(iv) $x^2-6xy+8y^2=x^2-2xy-4xy+8y^2$
$=x(x-2y)-4y(x-2)$
$=(x-4y)(x-2y)$

(v) $2x^2+3xy+y^2=2x^2+xy+2xy+y^2$
$=x(2x+y)+y(2x+y)$
$=(x+y)(2x+y)$

(vi) $3x^2+x-2=3x^2+3x-2x-2$
$=3x(x+1)-2(x+1)$
$=(3x-2)(x+1)$

(vii) $4x^2-8x+3=4x^2-2x-6x+3$
$=2x(2x-1)-3(2x-1)$
$=(2x-3)(2x-1)$

(viii) $4x^2-25y^2=(2x+5y)(2x-5y)$

(ix) $6x^2 - xy - 12y^2 = 6x^2 + 8xy - 9xy - 12y^2$
$= 2x(3x + 4y) - 3y(3x + 4y)$
$= (2x - 3y)(3x + 4y)$

3. (i) Using the difference of two squares:
$(x+4)^2 - (x-3)^2 = ((x+4) + (x-3))((x+4) - (x-3))$
$= (x + 4 + x - 3)(x + 4 - x + 3)$
$= (2x + 1) \times 7$
$= 7(2x + 1)$

(ii) Using the difference of two squares:
$(2x - y)^2 - (x + 3y)^2 = \big((2x - y) + (x + 3y)\big)\big((2x - y) - (x + 3y)\big)$
$= \big(2x - y + x + 3y\big)\big(2x - y - x - 3y\big)$
$= (3x + 2y)(x - 4y)$

4. (i) $\dfrac{2a^2b}{4ab^2} = \dfrac{\cancel{2} \times \cancel{a} \times a \times \cancel{b}}{^2\cancel{4} \times \cancel{a} \times \cancel{b} \times b} = \dfrac{a}{2b}$

(ii) $\dfrac{12p^2qr^3}{9pq^2r} = \dfrac{^4\cancel{12} \times \cancel{p} \times p \times \cancel{q} \times \cancel{r} \times r \times r}{^3\cancel{9} \times \cancel{p} \times \cancel{q} \times q \times \cancel{r}} = \dfrac{4pr^2}{3q}$

(iii) $\dfrac{x^2y + xy^2}{x + y} = \dfrac{xy\cancel{(x+y)}}{\cancel{x+y}} = xy$

(iv) $\dfrac{a}{2b} \times \dfrac{3bc}{a^2} \times \dfrac{a}{6c} = \dfrac{\cancel{a} \times \cancel{3} \times \cancel{b} \times \cancel{c} \times \cancel{a}}{2 \times \cancel{b} \times \cancel{a} \times \cancel{a} \times ^2\cancel{6} \times \cancel{c}} = \dfrac{1}{4}$

5. (i) $\dfrac{x^2 + x - 6}{x^2 - x - 2} = \dfrac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \dfrac{x+3}{x+1}$

(ii) $\dfrac{x^2 - 4x + 4}{x^2 + x - 6} = \dfrac{(x-2)^{\cancel{2}}}{(x+3)\cancel{(x-2)}} = \dfrac{x-2}{x+3}$

(iii) $\dfrac{x^2 + x - 2}{x^2 + 4x + 3} = \dfrac{(x+2)(x-1)}{(x+3)(x+1)}$ - cannot be simplified.

(iv) $\dfrac{4x^2-1}{4x^2-4x-3}=\dfrac{\cancel{(2x+1)}(2x-1)}{\cancel{(2x+1)}(2x-3)}=\dfrac{2x-1}{2x-3}$

(v) $\dfrac{2x+3}{3x+1}\times\left(3x^2-2x-1\right)=\dfrac{2x+3}{\cancel{3x+1}}\times\cancel{(3x+1)}(x-1)=(2x+3)(x-1)$

(vi) $\dfrac{x+2}{2x^2-x-1}\div\dfrac{x^2-x-6}{2x+1}=\dfrac{x+2}{(2x+1)(x-1)}\div\dfrac{(x-3)(x+2)}{2x+1}$

$=\dfrac{\cancel{x+2}}{\cancel{(2x+1)}(x-1)}\times\dfrac{\cancel{2x+1}}{(x-3)\cancel{(x+2)}}$

$=\dfrac{1}{(x-1)(x-3)}$

6. (i) $\dfrac{2x}{5}+\dfrac{3x}{2}=\dfrac{4x}{10}+\dfrac{15x}{10}$

$=\dfrac{19x}{10}$

(ii) $\dfrac{3a}{4}-\dfrac{2b}{3}=\dfrac{9a}{12}-\dfrac{8b}{12}$

$=\dfrac{9a-8b}{12}$

(iii) $\dfrac{2x+1}{12}-\dfrac{x-2}{8}=\dfrac{2(2x+1)}{24}-\dfrac{3(x-2)}{24}$

$=\dfrac{4x+2-3x+6}{24}$

$=\dfrac{x+8}{24}$

(iv) $\dfrac{3x+4}{2x}-\dfrac{5x+6}{3x}=\dfrac{3(3x+4)}{6x}-\dfrac{2(5x+6)}{6x}$

$=\dfrac{9x+12-10x-12}{6x}$

$=\dfrac{-x}{6x}$

$=-\dfrac{1}{6}$

(v) $\frac{1}{p}+\frac{1}{q}=\frac{q}{pq}+\frac{p}{pq}$

$=\frac{q+p}{pq}$

(vi) $\frac{a}{2b}+\frac{5b}{3a}=\frac{3a^2}{6ab}+\frac{10b^2}{6ab}$

$=\frac{3a^2+10b^2}{6ab}$

(vii) $\frac{3}{2x+1}-\frac{2}{x-1}=\frac{3(x-1)}{(2x+1)(x-1)}-\frac{2(2x+1)}{(x-1)(2x+1)}$

$=\frac{3x-3-4x-2}{(2x+1)(x-1)}$

$=\frac{-x-5}{(2x+1)(x-1)}$

(viii) $\frac{2x}{x-2}-\frac{x+1}{x+3}=\frac{2x(x+3)}{(x-2)(x+3)}-\frac{(x+1)(x-2)}{(x+3)(x-2)}$

$=\frac{2x^2+6x-(x^2-x-2)}{(x-2)(x+3)}$

$=\frac{x^2+7x+2}{(x-2)(x+3)}$

7. (i) $ax+b=c$

$ax=c-b$

$x=\frac{c-b}{a}$

(ii) $p-qx^2=r$

$p=r+qx^2$

$p-r=qx^2$

$\frac{p-r}{q}=x^2$

$x=\sqrt{\frac{p-r}{q}}$

(iii) $\sqrt{\frac{x}{s}} = t$

$\frac{x}{s} = t^2$

$x = st^2$

(iv) $a - \frac{b}{x} = c$

$a = c + \frac{b}{x}$

$a - c = \frac{b}{x}$

$x(a - c) = b$

$x = \frac{b}{a - c}$

(v) $px + q = a - bx$

$px + bx + q = a$

$px + bx = a - q$

$x(p + b) = a - q$

$x = \frac{a - q}{p + b}$

(vi) $y = \frac{1}{w(z - x^2)}$

$wy(z - x^2) = 1$

$z - x^2 = \frac{1}{wy}$

$z = \frac{1}{wy} + x^2$

$z - \frac{1}{wy} = x^2$

$x = \sqrt{z - \frac{1}{wy}}$

8. (i) $x^2 + 2x - 3 = (x + 1)^2 - 1^2 - 3$

$= (x + 1)^2 - 4$

(ii) $x^2 - 6x + 1 = (x - 3)^2 - 3^2 + 1$

$= (x - 3)^2 - 8$

(iii) $x^2+x+1=\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1$

$=\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+1$

$=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

(iv) $-x^2+5x=-\left(x^2-5x\right)$

$=-\left(\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2\right)$

$=-\left(x-\frac{5}{2}\right)^2+\frac{25}{4}$

(v) $2x^2+4x+3=2\left(x^2+2x\right)+3$

$=2\left((x+1)^2-1^2\right)+3$

$=2(x+1)^2-2+3$

$=2(x+1)^2+1$

(vi) $3x^2+8x-2=3\left(x^2+\frac{8}{3}x\right)-2$

$=3\left(\left(x+\frac{4}{3}\right)^2-\left(\frac{4}{3}\right)^2\right)-2$

$=3\left(x+\frac{4}{3}\right)^2-3\times\frac{16}{9}-2$

$=3\left(x+\frac{4}{3}\right)^2-\frac{16}{3}-2$

$=3\left(x+\frac{4}{3}\right)^2-\frac{22}{3}$

9. Method 1

$$\frac{x^2+6x+8}{2x^2+7x-4}=3$$

$$\frac{(x+4)(x+2)}{(x+4)(2x-1)}=3$$

$$\frac{x+2}{2x-1}=3$$

$$x+2=3(2x-1)$$

$$x+2=6x-3$$

$$5=5x$$

$$x=1$$

Check in original equation: $\frac{1+6+8}{2+7-4}=\frac{15}{5}=3$

Method 2

$$\frac{x^2+6x+8}{2x^2+7x-4}=3$$

$$x^2+6x+8=3(2x^2+7x-4)$$

$$x^2+6x+8=6x^2+21x-12$$

$$0=5x^2+15x-20$$

$$x^2+3x-4=0$$

$$(x+4)(x-1)=0$$

$$x=1,-4$$

Check in original equation. -4 leads to 0/0 so x=1.

10. Method 1

$$(m+n)^2+m^2-n^2=(m+n)^2+(m+n)(m-n)$$

$$=(m+n)(m+n+m-n)$$

$$=(m+n)(2m)$$

This has 2 as a factor so it must be even.

Method 2

$$(m+n)^2+m^2-n^2=m^2+2mn+n^2+m^2-n^2$$

$$=2m^2+2mn$$

$$=2m(m+n)$$

This has 2 as a factor so it must be even.